Dynamic Loading Tests: A State of the Art of Prevention and Detection of Deep Foundation Failures

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ABSTRACT. While observations during pile installation have been used for centuries to produce a quality deep foundation, using electronic measurements routinely has only become possible with modern sensors and computers. The beginning of this development took place more than 50 years ago in the 1960s. While the traditional and one of the most important methods of deep foundation quality assurance is the static loading test, it is now often partially or completely replaced by the Dynamic or High Strain Dynamic Loading Test. After a brief review of all available Quality Assurance and Quality Control methods, the paper describes the High Strain Test procedure, its theoretical background, equipment, software and standards. Two examples, one for driven and one for a cast-in-situ pile, demonstrate the benefits and limitations of the available methods.

1. INTRODUCTION

All deep foundation types, whether prefabricated and driven or cast-in-situ after drilling will be called a "pile" in this paper. Deep foundations are needed where surficial soils have insufficient strength to support a building, bridge or other structure. Because of their importance for adequate performance and because of their cost, deep foundation specifications and building codes increasingly demand thorough testing. As a reward for a thorough testing regime, modern codes reward the project with reduced factors of safety (AASHTO, 2014). The economic impact from the savings realized with reduced factors of safety can be very significant (Likins 2015).

1.1 Static Loading Methods

Tests were always performed, most notably the static top-down loading test, to verify adequate foundation performance. The load carrying capacity of both driven piles and bored piles has traditionally been evaluated by static load tests (SLT). Using either dead weights or reaction piles, the test pile is loaded by a hydraulic jack pressing against a reaction frame either in compression (ASTM D1143) or in tension (ASTM D3689). For larger test loads ASTM D1143 requires an instrumented load cell.

Although the top load static test is the reference test against which all other tests methods are judged, the test's shortcomings are numerous: The test is expensive and time consuming, contains many potential error sources and is non-unique as far as interpretation, as has been widely documented in the literature (i.e., Fellenius, 1990). Moreover, when it comes to large loads, the test can be very dangerous due to high "dead" loads (kentledge) or locked-in energies when elastic reaction systems are in use.

For large cast-in-situ piles with high capacities the safer and usually more economical bi-directional test has been developed (Elisio Da Silva, 1983 and Osterberg, 1984) and is being performed with increasing frequency around the world. It is particularly useful for large cast-in-situ piles. Rather than placing the jack at the top of the pile, the jack is attached to the reinforcing cage and inserted in the pile. While the embedded jack is often placed near the bottom of the shaft, it can also be placed at any location along the pile length. When pressurized it then exerts a downward force below the jack (resisted by end bearing and any shaft resistance below the jack) and an upward force above the jack (resisted by the pile weight and the shaft resistance above the jack. The upward and downward force components are combined into an equivalent applied load using strain compatibility principles. Advantages over top loading tests are first that very high loads can be safely applied and secondly the more accurate determination of end bearing vs. displacement compared to any other loading method. Disadvantages are that the jack (or jacks for high loads) is not recovered, that no load cell can be reasonably utilized for improved accuracy, that it must be assumed that the shaft resistance is the same under upward and downward movements and that the separation of the upper and lower shaft sections near the toe cause a different stress and strain regime than under top loading, and that the pile top is at zero load during the testing (so structural integrity of the upper shaft is not evaluated as it would be in the top-down test).

1.2 Dynamic Formulas

For driven piles, dynamic formulas have been widely accepted because of their simplicity; their benefits, if any, and limitations have been described by Allin et al. (2015) and Likins et al. (2012a). Dynamic formulas use generally only an assumed energy and an observed blow count to assess pile bearing capacity. Their use is, therefore, limited to driven piles. Their simplicity is often given as a reason for their use, however, that simplicity is also the reason for poor correlations with static test results which a myriad of versions and factor adjustments could not overcome, thus requiring uneconomically high factors of safety. In addition these formulas do not allow for stress calculations.

1.3 Pile Dynamic Analysis and Simulation Methods

A numerical solution of the underlying wave equation became practical with the availability of the electronic computer. Great interest among both construction and academic professionals was generated by Smith (1960) in the United States. At about the same time in Europe, Fischer (1960), for example, developed his graphical solution of the wave equation which he applied to a number of pile driving problems and which was related to software developed by others such as Meunier (1984). Any of these impact simulation programs proved to be much more accurate than the traditional dynamic formulas and not only provided a relationship between the number of blows per unit penetration (blow count) and bearing capacity, but also a reasonably accurate prediction of dynamic pile stresses. Among the additional options included in these programs was the driveability analysis which became particularly popular since it simulated not only what happens under one hammer blow, but rather what can be expected during the complete pile installation process using diesel, hydraulic, air, steam or even vibratory hammers (e.g., Rausche et al., 2009).

1.4 The Case Method of Dynamic Pile Testing

Beginning in the late 1950s measurements which could be routinely performed on construction sites were developed at Case Western Reserve University (Goble and Rausche, 1970a); Hussein and Goble (2004) briefly described these developments. Interpretation of the measured force and velocity signals were based on closed-form solutions of the wave equation (Goble and Rausche, 1970b). This approach was so successful, particularly after being introduced in Sweden and from there around the world, that the first Stress Wave Conference was held in Gothenburg in 1980 with repeat conferences every 4 years. A wealth of papers describing the advancement of the dynamic pile testing methods has been generated by these conferences. Similar development efforts were also made in Europe, see for example Beringen et al. (1980). The Case Method is further described below.

The sensors developed during the 12 years of research at Case Western Reserve University (CWRU) included reusable strain transducers which would be bolted to the side of the pile. The strain signal was then multiplied with the pile's elastic modulus and cross sectional area to yield the pile force. Alternatively a so-called "top transducer" was used (Goble and Rausche, 1970a) which could be calibrated in a universal testing machine and avoided concrete modulus uncertainties when cast-in-situ piles were tested. The top transducer was basically an instrumented, short section of heavy-wall pipe. Velocity was measured using accelerometers whose signals were integrated to yield velocity. While various changes in sensor technology improved significantly, the quality of the signals recorded and evaluated, the basic measurement systems are still the same as developed during the original research. ASTM D4945, "Standard Test Method for High Strain Dynamic Testing of Deep Foundations", specifies how the dynamic tests have to be performed. Similar standards now exist in many other countries or regions such as Australia, Brazil, China, and Europe.

For the calculation of pile bearing capacity two approaches were chosen; one was a simple equation that could be solved by computer between hammer blows and one a signal matching approach which would be more computer time extensive. The simple equation was first based on a rigid pile model with resistance calculated at the time of zero velocity used (Goble and Rausche, 1970a). Today this approach is also called an unloading point method and is applied to the interpretation of the Rapid Load Testing Method (Middendorp et al., 1992). However, it soon became clear that a closed form solution to the wave equation such as has been described by Timoshenko and Goodier, (1951) would be more accurate (Rausche et al., 1972); it was called the "Case Method Equation".

The signal matching procedure developed by the Case team, called CAPWAP®, relied first on Smith's numerical analysis approach (Rausche et al., 1972); it was later replaced by the characteristics solution to the wave equation (Rausche, 1988) called CAPWAPC (today it is again referred to as CAPWAP). Today a fast automatic signal matching program, iCAP (Likins, et al., 2012b) is also available for uniform, driven piles. While not taking the place of the more accurate and powerful CAPWAP program, iCAP can replace the Case Method with the advantage that it does not require an estimated damping factor. Also, results from iCAP are unique, i.e., independent of user experience. In the 1970s other formulas were also developed for the assessment of hammer performance, pile stresses and pile integrity from pile top force and velocity as discussed below.

2. PREVENTION AND DETECTION OF DEEP FOUNDATION FAILURES

Two different deep foundation failures have to be distinguished Geotechnical and Structural failures. The former is best detected or prevented by loading tests and, for driven piles, dynamic monitoring. The latter is generally only measured and assessed by dedicated integrity tests. While the static loading test is least suitable for detecting structural deficiencies unless they cause the pile to catastrophically fail during the loading, the dynamic loading test has a better chance to detect defects as will be discussed in the Section on dynamic monitoring. The following describes the most common and recognized integrity test methods and lists additional integrity tests that are less commonly used after mentioning a few construction monitoring methods which can help prevent pile integrity problems.

2.1 The Pile Installation Recorder

This device measures, during the construction of augered-cast-in-place (ACIP) or Continuous Flight Auger (CFA) piles, the pumped concrete volume as the auger is retracted. While volume measurements sometimes only rely on the counting of pump strokes, the preferred and more accurate means is uses the magnetic flow meter. The basic result of this monitoring is a concrete volume vs. depth record, sometimes augmented by pump pressure and auger torque (Brown et al., 2007) While the volume vs. depth record is an important document (Piscsalko et al, 2004) it is even more important that the operator can view the measurements in real time and take corrective action should the incremental pumped volume with depth fall below requirements.

2.2 Drilled Hole Inspection by Caliper

After completion of the drilling and prior to placing concrete in the bored pile, the shape or profile of the hole can be measured with a so-called caliper. Assuming that no further changes in the hole occur after the measurement and during concreting, the shape of the finished pile has been established. Some caliper devices use mechanical arms while more modern devices use an ultrasonic technique.

2.3 The Shaft Inspection Device (SID)

For bored piles, when end bearing is considered in the design, the condition of the bottom of the drilled hole is important and must be "clean", meaning loose sediment removed, so that end bearing is activated at a relatively small displacement rather than first compressing a weak debris layer. This is particularly important in rock sockets. While not directly measuring pile capacity, the SID is being used with a thin measuring rod penetrating the soft sediments. The rod penetration into the sediment is viewed with a remote camera which also displays the cleanliness of the bottom surface.

2.4 The Quantitative Inspection Device (SQUID)

A more advanced inspection tool, the SQUID, actually measures the force and distance required to penetrate any potential debris layer and also the resistance in the bearing layer using one or more instrumented cones (Fig. 1). The device is conveniently attached to the Kelly bar or drill stem and quickly inserted in the water or slurry filled or dry hole. The whole process typically takes less than 15 minutes. When the hole is confirmed as clean and the required penetrometer force is adequately confirmed, the pile can be concreted and the end bearing included in the design. Such a device can be particularly cost effective to minimize the depth of a rock socket by determining when the rock is of sufficient strength.

2.5 Low Strain Integrity Testing, (Pulse Echo Testing, PIT)

One of the earliest methods, and one which has great similarities with the high strain method, is low strain integrity testing (Rausche et al, 1988). The method of data collection is specified in ASTM D5882. The method can be quickly applied and testing 100 piles in one day is not impossible. Higher testing rates may lead to poor data quality.

Once concrete of the pile has hardened sufficiently, the pile top is struck by a hand-held hammer which generates a force wave that travels down the pile shaft, reflects off the pile toe or other cross section changes and then propagates back to the pile top. An accelerometer, attached to the pile top, measures both the input and reflections. The acceleration is integrated to velocity and various enhancements to the velocity record are made to facilitate data interpretation which is based on basic wave propagation theory (Likins and Rausche, 2000). For example, Fig. 2 shows two records of pile top velocity vs. time enhanced by an exponential amplification function to compensate for signal losses due to soil and pile material damping. The top signal shows a strong positive reflection at a time corresponding to the designed 25 m length of the pile. The bottom graph, on the other hand, displays a smaller reflection corresponding to a 15 m depth suggesting an anomaly of pile size or concrete quality 10 m above the pile toe.

Occasionally a different type of low strain data interpretation, referred to as the Transient Response Method, (Rausche et al., 1991), is employed. Actually, it requires that, in addition to the velocity, the impact force of the hand held hammer is measured. It calculates by Fourier Transfer the Mobility of the pile in the frequency domain thereby yielding, for example, a pile stiffness value.

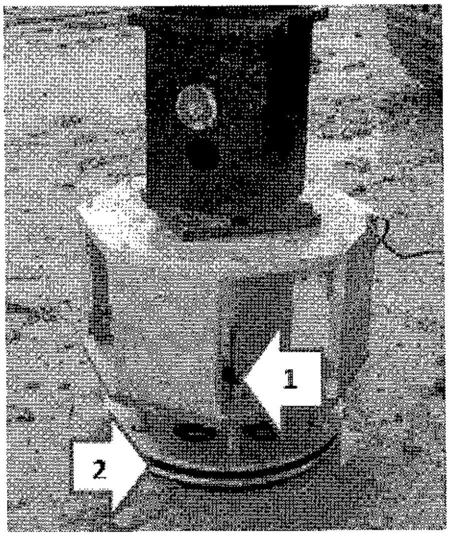

Fig.1. The SQUID prior to insertion in the drilled hole: (1) calibrated force transducer; (2) displacement measuring plate.

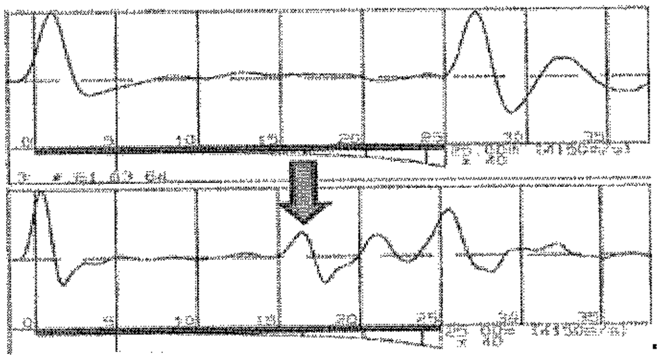

Fig. 2. Presentation of PIT records (a) top intact pile and (b) defective pile

2.6 Cross Hole Sonic Logging (CSL)

Crosshole Sonic Logging (Likins et al, 2004), commonly called "CSL", requires installation of at least two water filled access tubes in the pile so that concrete quality can be assessed by measuring the wave travel time in the concrete between a transmitter in one tube and a receiver in the other tube. Typically one tube for each 300 mm of pile diameter is required. The method is standardized in ASTM D6760. From the "First Arrival Time" (FAT) of the signal and the spacing between access tubes, the wave speed can be calculated.

The wave speed magnitude is considered a measure of the concrete quality. FAT decreases of more than 20 or 30% are generally considered an anomaly of concern. CSL is not sensitive to the surrounding soil and is not limited by pile length. Unfortunately, it also gives no indication of the quality or quantity of the concrete outside the reinforcing cage and sometimes may falsely indicate problems due to "debonding" of the access tubes from the surrounding concrete which inhibits wave propagation. If low wave speeds are detected in several scans at the same cross section, then a "tomography" analysis (Likins et al., 2007) can be useful to estimate the extent of the anomaly (Fig.3).

2.7 Thermal Integrity Profiling

Cement produces heat during the curing process. This phenomenon is the basis for Thermal Integrity Profiling of the entire cross section of bored piles (Piscsalko et al., 2013). During the curing of a bored pile, the center of the pile has the highest temperature while the perimeter has the lowest temperature since it is adjacent to the soil and the heat is flowing from the pile into the soil. The more cement content in a concrete mix, the higher the temperature created and, therefore, the lower the temperature the lower the concrete quality or pile size. The Thermal Integrity Profiling method procedures are governed by ASTM D7949. While temperature can be sensed by infrared probes in access tubes, it is more convenient to measure the temperature by attaching cables with thermal sensors to the reinforcing cage. One such instrumented cable is installed equidistantly around the cage for each 300 mm of pile diameter. The average temperature of the shaft can be correlated to the effective average shaft radius. Local deviations from the average shaft temperature can then be related directly to deviations from the average shaft radius, allowing for a complete evaluation of the entire cross section including the concrete cover outside the reinforcing cage. Fig. 4 shows a 3D image calculated from temperature measurements. It should be noted that the test has to be performed while the concrete cures which makes for a quick turnaround of results (often be completed within 24 hours of casting concrete), saving valuable construction time.

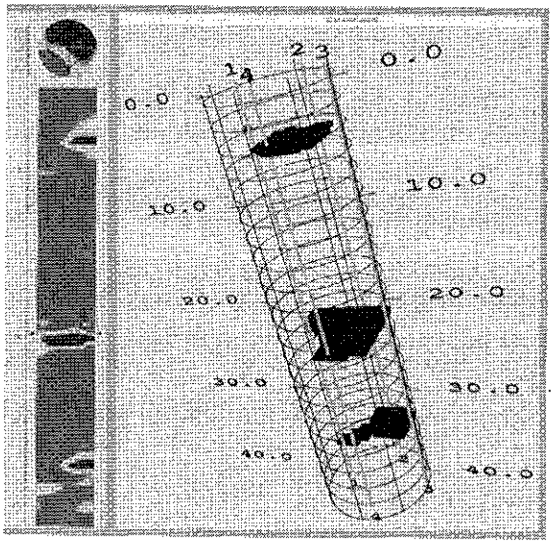

Fig. 3: Signal arrival times in a shaft with 4 tubes from a CSL tomography; the dark colors indicate low arrival times (FAT).

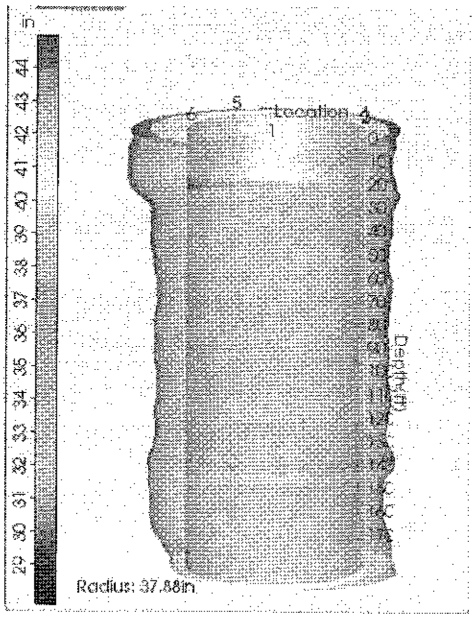

Fig. 4: 3D image right from thermal measurements.

2.8 Other Integrity Test Methods

There are other methods which have been reported on and which are occasionally used depending on the purpose and or preference of the specifier. The following lists first three of more successfully used methods and then two additional ones that are less frequently used.

1. The Gamma-Gamma Method helps identify concrete cover defects by measuring concrete density around the inspection tubes and which is used in conjunction with CSL testing.
2. The Length Inductive Test Method (LITE) which also requires a borehole next the pile to be tested for length; in this case the pile has to be made of metal.
3. The Parallel Seismic Method which requires a borehole next to the pile, an accelerometer in the borehole and a light hammer impact on the pile top for pile length determination.
4. The Bending Wave method which applies impacts to a pile (typically a timber pile under a structure) perpendicular to its axis (there are similarities with the Low Strain Method).
5. The Vibration Method which applies a variable frequency vibration to the pile top and which is evaluated in the frequency domain similar to the Transient Response Method.

3. DYNAMIC PILE ANALYSIS AND HIGH STRAIN TESTING

3.1 Wave Equation Analysis

Dynamic formulas do not consider pile type or dimensions, hammer configuration or actual hammer efficiency (Allin et al., 2015), driving system components, or soil type and profile and they cannot calculate stresses; these limitations gave rise to the development of the wave equation approach by Smith (1960). Among several proprietary computer programs now in existence, GRLWEAP is probably most widely used and is further described in Rausche et al., 2004 and Hannigan et al., 2016.

In this "wave equation" approach, the hammer and pile are modeled by a series of masses and springs and an initial impact velocity is assigned to the ram segments. The soil is modeled with both springs, representing the static capacity, and dampers, representing viscous effects. Using short time increments the resulting forces in the springs and motions of all pile segment masses are computed as time progresses. Thus, the maximum stresses at every location in the pile can be evaluated for any modeled situation, and the final net displacement calculated for an assumed or calculated (by standard static methods) capacity. Generally several capacities are assumed and the corresponding blow counts computed and the resulting relation of input capacities to computed blow counts is known as a bearing graph. Given accurate soil information, capacity can be estimated for any depth by static analysis and the installation analyzed at various depths of penetration as a check for pile driveability, i.e., to prove that the hammer is capable of installing the pile to the desired depth or capacity while keeping driving stresses within acceptable bounds.

While the resistance distribution is determined by standard static geotechnical methods, such as the alpha or beta methods, additional dynamic soil resistance parameters are needed. They include for the static resistance the quake (equal to the ultimate resistance divided by the soil stiffness). A dynamic resistance parameter, called damping factor, also has to be specified.

Additionally, for the so-called driveability analysis, which determines an estimated blow count vs. depth, the so-called soil setup factor has to be input for each soil layer. A driveability method popular for monopole installations, called friction fatigue, requires an additional shape factor which describes the resistance distribution during pile driving (Alm and Hamre, 2001).

These parameters as well as a nearly complete hammer and driving system input help is provided by the GRLWEAP program.

The wave equation analysis has been widely accepted as a valuable tool in assessing compatibility of the hammer with the pile for a specific soil profile, and to evaluate the driving stresses in such scenarios to prevent pile structural damages. While not a "test" in itself, combining the results of a wave equation analysis with the observed pile penetration per blow (set) or blow count taken at the end of a pile installation or during a restrike test allows for assessing the bearing capacity of the pile. An input screen of the GRLWEAP program is shown in Fig. 5 representing data for a 1724x35.4 mm open ended pipe pile of 47.6 m length, driven through water and then sand into calcarenite rock, reaching a final depth below mudline of 19.1 m. The hydraulic hammer, an IHC 200S, was run at a reduced energy of 72% at the end of driving. A restrike test conducted 10 days later was run with an energy setting of 57%. For these two energy levels the wave equation analysis with standard soil parameters for granular soils produced the bearing graphs shown in Fig. 6. Entering these bearing graphs with respective end-of-drive and restrike blow counts of, respectively, 54 and 125 blows for 250 mm penetration (corresponding to 4.6 mm and 2 mm set per blow) the wave equation would predict capacities of 12,000 and 15,700 kN. The bearing graphs also indicate refusal capacities between 22,000 and 24,000 kN. The associated calculated compressive stresses were 146 and 133 MPa. These results will be further discussed below.

While this program was developed for impact driven piles and has become an indispensable tool for pile installation preparation and construction control, today it is also used to check the driveability of vibratory driven piles (Rausche and Beim, 2012) and for the preparation of dynamic loading tests even on cast-in-situ piles (Hussein et al., 1996).

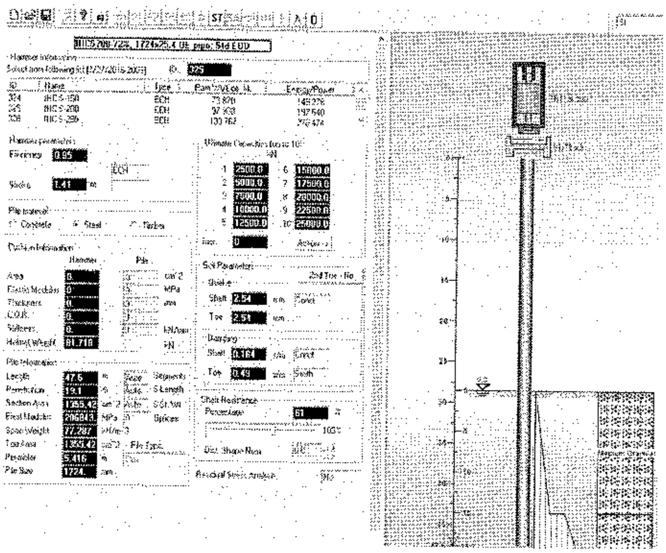

Fig. 5. Example Bearing Graph Input.

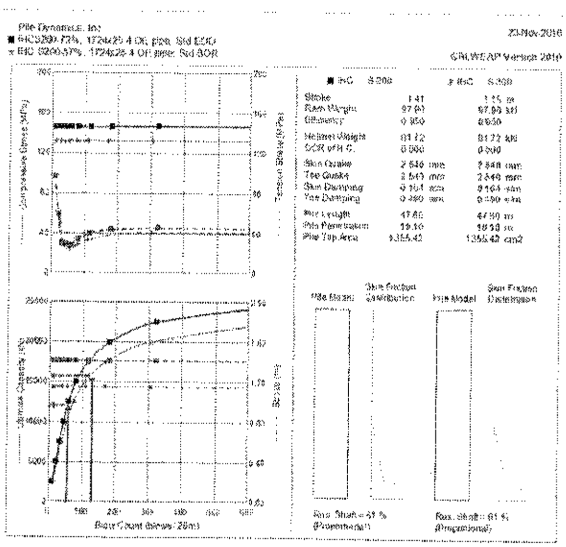

Fig. 6. Example Bearing Graph Output

3.2 High Strain Testing

3.2.1 Dynamic Monitoring

For impact driven piles, an installation log showing hammer energy setting and observed blow count is still an indispensable quality control tool, however, in addition electronic measurements are frequently taken for a quantification and more accurate assessment of pile driving equipment performance, pile stresses, pile integrity and pile bearing capacity. Fig. 7 (left) shows photos of strain and acceleration sensors attached to an H-pile; strain is converted to force by multiplication with cross sectional area and elastic modulus. Velocity is calculated from acceleration by integration over time. Fig. 7 (right) shows a force transducer on top of a drilled shaft; it provides a direct measurement of force. The force and velocity data are analyzed in real time by closed-form solutions of the wave equation, most notably the "Case Method" initially developed during the CWRU research and later modified to some degree (Rausche et al, 1985). The closed-form Case Method Equation allows for immediate analysis on site blow-by-blow to calculate the static soil resistance component, R, using Equation 1.

$$R = (1-J)(F(t_1) + Z\,V(t_1))/2 + (1+J)(F(t_2) - Z\,V(t_2))/2 \qquad (1)$$

where

$F(t_1)$ is the measured force at a chosen time, t1,
$V(t_1)$ is the measured pile velocity at the same time t1
$t_2 = t_1 + 2L/c$
L is the length of the pile below the sensors
c is the pile material wave speed and
$Z = EA/c$ is the pile impedance
E is the pile elastic modulus,
A is the pile cross sectional area and
J is the damping factor which is related to the soil type, typically ranging from 0.4 for coarse grained soils to 1.0 for cohesive soils. It is normally determined by CAPWAP analysis of one of the records obtained.

Equation (1) is evaluated for all times t_1 between impact and the end of each of the acquired records, i.e., of all monitored hammer blows. The maximum value of resistance, R_x, is considered the best estimate of the ultimate soil resistance. The damping factor, J, has to be estimated based on soil grain size or determined by correlation with either CAPWAP analyses or static load tests.

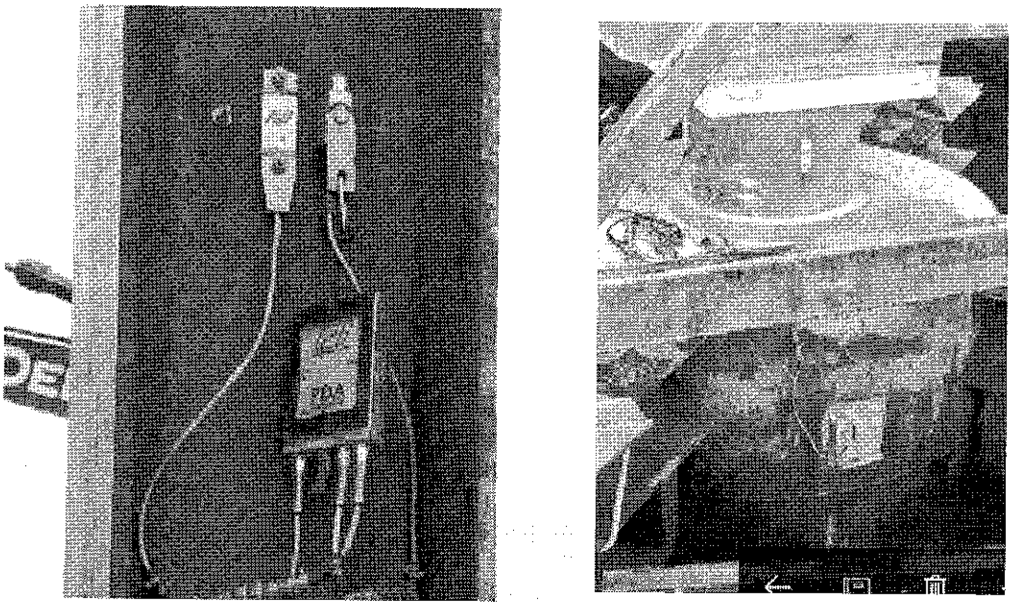

Fig. 7. Left: Accelerometers (right) and strain transducers (left) and data transmitter all attached to an H-pile; Right: Instrumented, cushioned "top transducer" on bored pile.

Another convenient formula allows for calculation of the energy transferred to the pile:

$$E(t) = +F(t)\ V(t)\ dt \tag{2}$$

The maximum value of E(t) is often referred to as EMX or ENTHRU and is a valuable hammer performance indicator. The ratio of EMX to rated energy, E_r, is called transfer ratio (also transfer efficiency or global efficiency). Experience has shown that certain types of hammers such as diesel, air/steam or hydraulically powered ones, have certain acceptable ranges of energy transfer for certain types of piles.

Stresses at the pile top are directly obtained from the strain measurements. Using one-dimensional wave propagation theory, the average compression at the pile toe and the maximum tension at any location along the shaft can also be evaluated from the pile top measurements. Keeping these stresses below the recommended limits based on structural material properties reduces the possibility of pile damage (Hannigan et al., 2016).

The extent of damage and its location can be quantified using the so-called Beta Method (Rausche and Goble (1979), Rausche et al., 1988 and Likins and Rausche (2014)). This method, applicable to uniform piles, is based on the wave propagation based theory and basically calculates an integrity indicator less than 100% if a tension wave reaches the pile top prior to the arrival of the stress wave from the pile toe. Clearly, this method has a great similarity to damage detection by the Low Strain method however, rather than investigating only the velocity record for reflections, the force signal now serves as a reference line. Fig. 8 shows two records of a 41 m long 380 mm square section concrete pile. The records consist of force and velocity times pile impedance (Z=EA/c, see Eq. 1 above); the product of velocity and impedance has the units of force. The vertical lines in these records indicate the onset of impact, t_o, and the onset of the wave return from the pile toe, i.e., t_o plus 2 times the pile length below sensors divided by the wave speed (2L/c).

The upper graph shows the force slightly increasing relative to velocity in the first 2L/c time period and the difference F minus VZ is a measure of the shaft resistance. In contrast, the lower graph shows a relative velocity increase about L/c after impact which can be interpreted as a signal from mid-pile length. This is indicated by a dashed vertical line. Since only a reduction in pile impedance can be responsible for such a record feature (velocity increasing relative to force) it must be concluded that the pile is damaged at mid-length.

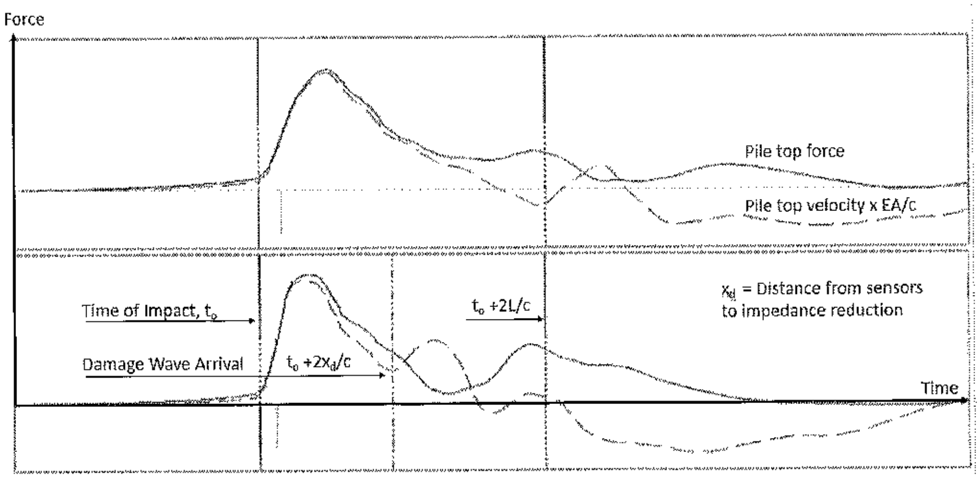

Fig. 8: Pile top force (solid) and velocity (dashed) records of a pile tested before (top) and after damage occurred (bottom).

Dynamic monitoring results for each hammer impact are conveniently plotted either vs. pile penetration depth or vs. blow number. These results are instructive and may be used, for example, to optimize the driving process. Fig. 9 shows for the 1724x25.4 mm open ended pipe pile, discussed above, pile top and bottom stresses (left), transferred energy and blows/minute (center) and blow count plus Case Method resistance for a damping factor J=0.5 (right). This is a comprehensive summary of results for every hammer blow and would, for example, indicate that the hammer energy setting should be reduced for areas with high stresses or that a reduced pile length is acceptable, if capacity is reached earlier than originally anticipated. Also, this information can be conveniently used to performed so-called refined wave equation analyses, i.e., wave equation analyses with hammer, driving system and soil parameter input values adjusted for energy, stress and blow count results matching the field observations.

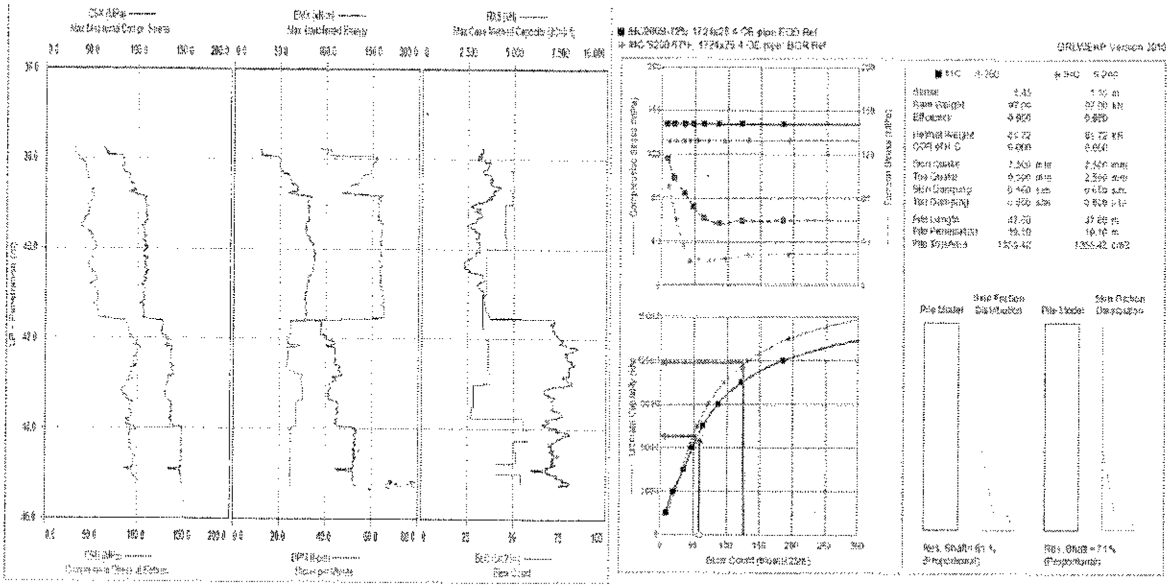

Fig. 9. Left: Pile driving monitoring results for the example case; Right: Wave equation bearing graph using dynamic loading test results for refinement.

3.2.2 Dynamic Loading Tests

Each hammer blow applied to a pile loads the pile during a short time period and activates soil resistance forces. The force and velocity data recorded under an impact loading can, therefore, be interpreted as a very quick loading test. Both driven piles and cast-in-situ piles can be tested in that manner (Rausche and Seidel, 1984 and Seidel and Rausche, 1984).

Since changes in the soil occur during pile driving, the end of driving blows may not be indicating the soil resistance that a static loading test would or what would be present under long term loading. Both soil resistance increases (soil setup) or decreases (relaxation) with time after pile installation have been observed. While for monitored and impact driven piles the end-of–driving (EOD) data is valuable for quality assurance and control purposes, it is, advisable to perform a restrike test after a certain, soil-type dependent waiting time. The beginning of restrike (BOR) data are usually closely related to the long term bearing capacity. During restrike testing it is possible that soil setup is so high that the installation pile hammer may not enough energy to move the pile under a single blow sufficiently (at least 2 mm) to mobilize the available ultimate soil resistance. In that case only a lower bound of the available soil resistance, or "proof load", would be calculated from the pile top measurements. To remedy this problem a high energy drop hammer is often mobilized as would also be necessary when cast-in-situ piles are tested. Another remedy would be the so-called superposition, i.e., combining EOD end bearing with BOR shaft resistance (Hussein et al., 2002).

The dynamic loading can be performed in two ways; the first often uses a pile driving hammer whose driving system, in particular, the cushions have been designed for a quick and efficient loading with cushioning just enough to protect the pile from excessive stresses. If a drop hammer is used, the pile top cushion is designed so that a high energy transfer is possible. This test configuration is covered by ASTM D4945.

The second dynamic loading method is referred to as Force Pulse or Rapid Test (ASTM D7383); Fig. 10 shows different configurations of the test setup. Originally, this test was performed by burning a combustive fuel in a chamber located above a load cell on the pile top and below a heavy reaction mass. This system was called Statnamic testing (Bermingham and Janes, 1989). The force pulse thus generated lasted for about 100 ms or about 5 to 10 times longer than typical dynamic load tests. Also the rise time, i.e., the time it would take the force to reach its peak was significantly longer that of a dynamic test. It was later realized that a similar force pulse could be generated by a heavily cushioned drop hammer. For example, the Fundex Pseudo Static tester uses large steel springs as per Schellingerhout and Revoort (1996). Alternatively using a dynamic testing system with very thick plywood cushions as per Rausche et al. (2008) or thick sandwiches of synthetic material described by Miyasaka et al. (2009) would produce long force pulses. However, the rise time of the signals produced by the latter two loading systems is similar to those of the dynamic test.

For the force pulse test, typical required drop weights or reaction masses are 5 to 10 percent of the desired ultimate capacity while dynamic tests are usually done with 1% drop weights when testing piles on rock and 2% otherwise. The advantage of a relatively long duration pulse is that tension stresses in the pile are then of little concern. However, since the velocity of the pile under the force pulse is at or above 1 m/s, dynamic resistance forces and inertia forces must be considered, and unless the pile is further instrumented with strain gages, resistance distribution cannot be deduced from this test.

There are widely differing opinions on how to deduce the equivalent static test from the basic measurements, particularly in cohesive soils (e.g. Middendorp et al., 1992; Matsumoto, 1994; Hajduk, 2000; Schmuker, 2005; Weaver, 2010; Brown, 2013). If there is a significant net settlement (minimum 3 percent of pile diameter), then the soil resistance is considered as "fully mobilized" and is perhaps considered more reliable (Miyasaka et al, 2009). In addition to axial tests, the Statnamic device has been deployed to apply lateral impacts, which help model, for example, ship impacts and earth quake loads.

While the Case Method is a good first indication of the mobilized soil resistance, for dynamic load testing the force and velocity data are always analyzed in a rigorous manner by "signal matching". The most commonly used software is called CAPWAP which stands for the Case Pile Wave Analysis Program. It is practically a wave equation program, but without the need to modeling hammer and driving system since the hammer impact effect is known from the pile top force and velocity records. (Rausche and Goble, 1972) and years of experience have demonstrated good agreement with static loading test results (Likins and Rausche, 2004). The advantage over the Case Method, besides finding the damping factors, is the ability to calculate quakes and soil resistance distribution. The CAPWAP method can also be used when the rise time of the force pulse is short as in the system described by Miyasaka et al., (2009).

The long rise time force pulse tests are not easily analyzed by CAPWAP because of the lack of a clear wave front activating successively resistance along the pile. These measurements are generally interpreted (a) by defining the dynamic soil resistance at the time when the pile top starts to rebound or unload, which implies that then the velocity and therefore damping is zero, and (b) by multiplying the unloading force by a rate effect correction factor. This factor varies between 1.0 for sands and approximately 0.5 for highly cohesive soils.

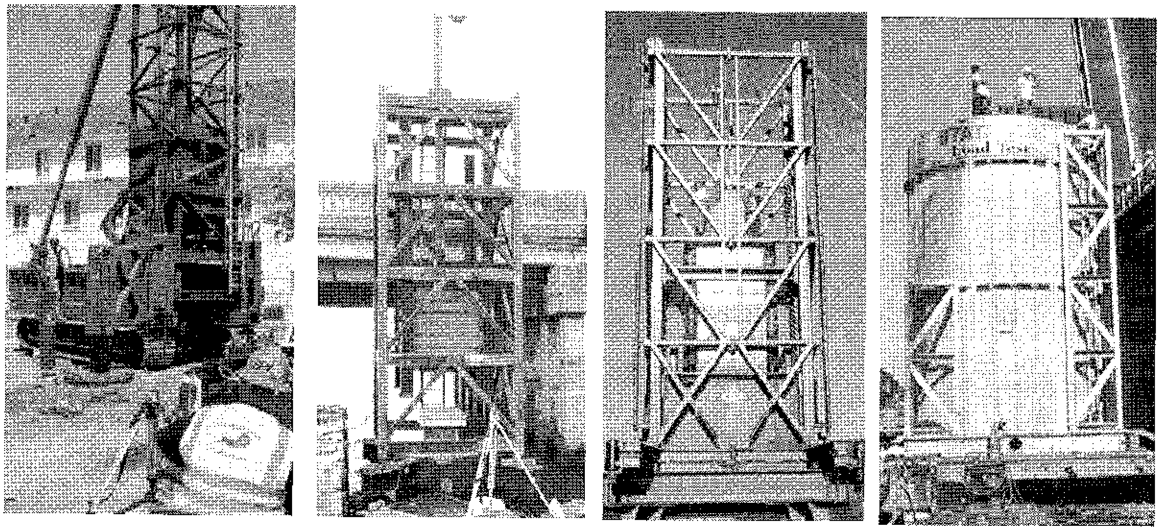

Fig. 10. From left to right: Fundex mass with heavy spring cushion; Cushioned dynamic test; Statnamic; Statnamic with gravel catch mechanism for heavy reaction masses.

4. EXAMPLE: HIGH STRAIN TESTING OF AN OPEN ENDED PIPE PILE

As an example of a CAPWAP result, the beginning of restrike (BOR) test of the wave equation and monitoring case discussed earlier is shown in Fig. 11. This example, the same that was used for demonstrating the wave equation approach, is particularly interesting because of occasional problems encountered with the dynamic testing of large diameter open ended pipe piles where full end bearing may not be mobilized in dynamic test due to soil plug slipping caused by inertia effects (Brown and Thompson, 2015). The CAPWAP summary page of Fig. 11 includes the record analyzed (upper right), the force signal match (upper left) the calculated resistance distribution (lower right) and the calculated load-displacement curve (lower left). The numerical values shown indicate a calculated total capacity of 11,700 kN with 3,400 kN end bearing. The end-of-driving (EOD) results, also analyzed by CAPWAP, indicated a lower capacity of 6,300 kN with 1,600 kN end bearing demonstrating an almost doubling of capacity during the 10 day waiting time between EOD and BOR.

Compared to the wave equation prediction shown above, obviously much lower capacity values have been calculated by CAPWAP. This difference is primarily due higher dynamic resistance factors: for the end-of-drive the CAPWAP quakes were 7.5 and 9.6 mm vs. the standard 2.5 mm used in the wave equation analysis. This is probably due to an increased pore water pressure which made the soil "spongy" or "bouncy" and thus more energy absorbing. For the restrike it was noticed that primarily the shaft damping was much higher than normally assumed (1.0 vs 0.16 s/m). Repeating the analyses, as so-called Refined Wave Equation Analyses, with the CAPWAP calculated quakes and damping for EOD and a strongly increased toe damping for BOR yielded the bearing graphs shown in Fig. 9, right, with capacities of 6,600 and 11,500 kN vs. 6,300 and 11,800 kN CAPWAP. (Note that adjustments to the CAPWAP soil parameters are usually needed to achieve blow count agreement, because of differences in the blow count calculation approaches of the two programs). Refined Wave Equation Analyses are useful when analyzing different situations at the same site based on a representative dynamic load test. Dynamic measurements and/or local experience are obviously invaluable for assessing proper dynamic resistance quantities and therefore better wave equation predictions.

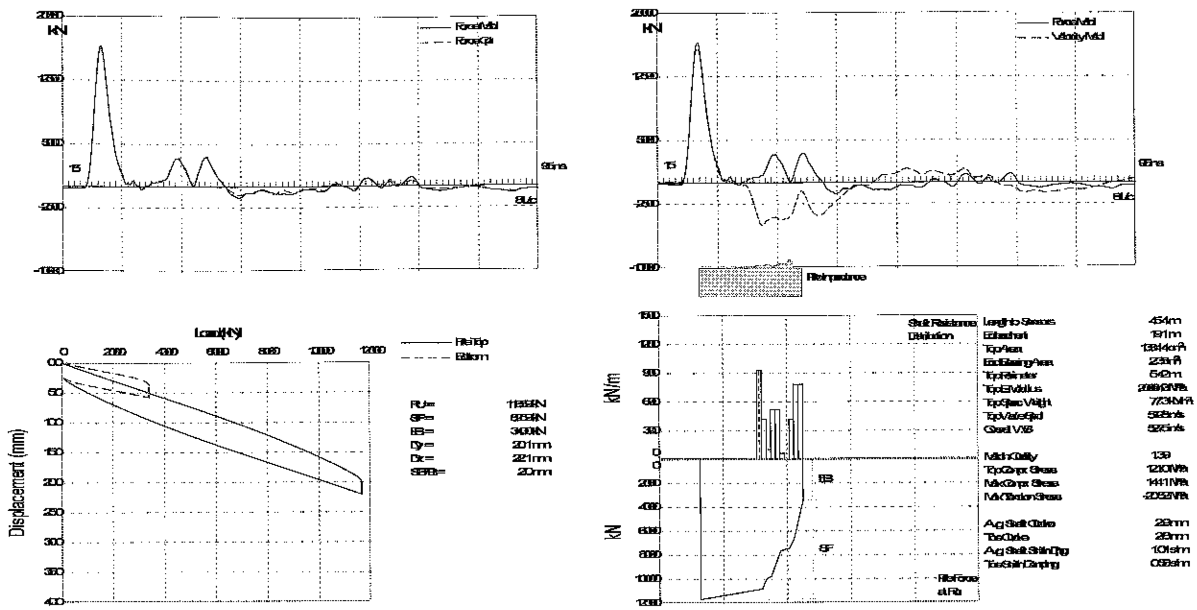

Fig. 11. CAPWAP summary for the 2nd restrike blow of the open ended pipe example.

The same open ended pipe pile was also subjected to an instrumented static loading test, 16 days after the restrike test., allowing for a comparison of statically and dynamically determined pile top load-displacement curves; the dynamic and static displacements are shown cumulatively in chronological testing order in Fig. 12, left; obviously the EOD test showed a relative lack of stiffness compared to the BOR and the static tests, apparently caused by the large quakes (equivalent to a reduced soil stiffness). The static test was conducted by applying 5 loading cycles. The envelope enclosing the five cycles indicated a 12,500 kN capacity for the Davisson failure criterion (Fellenius, 1990) which suggests a small, additional soil setup during the 16 day waiting period after the restrike test; in any event a good agreement with the BOR test was achieved. Instrumentation attached to the pile along its length also allowed for a comparison of the forces in the pile at the corresponding CAPWAP load level and the Davisson failure load. Obviously, the resistance distribution calculated from the dynamic test is similar but not very close to the statically determined one (Fig. 12, right).

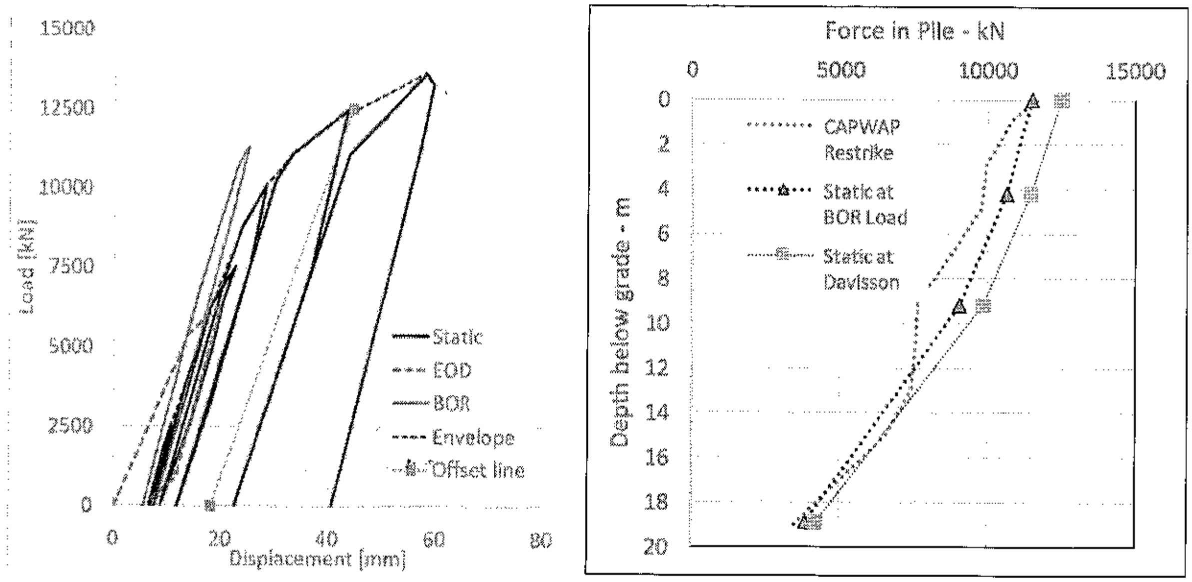

Fig. 12. Left: EOD, BOR and 5-cycle static load-displacement curves; Right: pile forces measured in pile at different load levels in static loading and from restrike test.

5. EXAMPLE: STATIC AND DYNAMIC LOADING TESTS OF A CFA PILE

Continuous Flight Auger (CFA) or Augered-Cast-in-Place piles (ACIP) are produced by drilling with an auger of the length of the pile and then pressure injecting grout from the bottom up through the hollow stem of the auger while the auger is extracted. While the monitoring of this installation process is valuable (see Pile Installation Recorder above) testing for geotechnical and structural sufficiency is equally important. Statically testing a few piles in the beginning of production piling may be good to proof the overall suitability of the pile for the project, however, usually only a few static loading tests are performed even though the number of piles on a site may be very large. Quick and relatively inexpensive dynamic testing is, therefore, frequently performed too, and agreement between static and dynamic test results has generally been very good. Also in recent years the use of embedded sensors has generated valuable comparison records which should, in the future, lead to even better agreement between dynamically and statically determined resistance distributions. Of course, for cast-in-situ piles, the question of elastic modulus and cross sectional area below grade is always a problem and may lead to inaccurate calculation of force from strain.

The following example shows results from both static and dynamic loading tests on a 44.5 m long CFA pile of 610 mm diameter which was auger-cast through deep soft cohesive soils into a sedimentary rock. Dynamic acceleration and force were measured on pile and with a top transducer, respectively. The pile had been instrumented with strain gaged sister bars attached to the center reinforcement bar of the pile at six locations. After the grout had sufficiently hardened, the dynamic load test was performed by dropping a 16-ton APPLE ram from various drop heights onto the lightly cushioned top transducer. The permanent displacement occurring under the dynamic impact analyzed was 3.2 mm. A few days later a static load test was performed.

For the interpretation of the static test on cast-in-situ piles, AASHTO specifies an offset criterion of 3.8 mm + D/120 for a 610 mm (or less) diameter (D) pile and 3.8 mm + D/30 for a 914 mm pile (or greater), with linear interpolation. In the present case the offset elastic line would intersect the slightly extrapolated static load test curve at a little less than 10,000 kN while the dynamic test activated 9,200 kN resistance. Practically the same static capacity value would be found if the Butler-Hoy criterion (Fellenius, 1990) were applied to the static load-displacement curve. The Butler-Hoy criterion is preferred for the interpretation of cast-in-situ piles in the building industry. In any event, the static and dynamic load-displacement curves agree quite well (Fig. 13, left) although the dynamic curve has a steeper slope at the origin. This difference is also evident in the resistance distriution curves from CAPWAP and static test (Fig. 13, right). It may be explained by a progressive failure of the upper shaft resistance during the static loading.

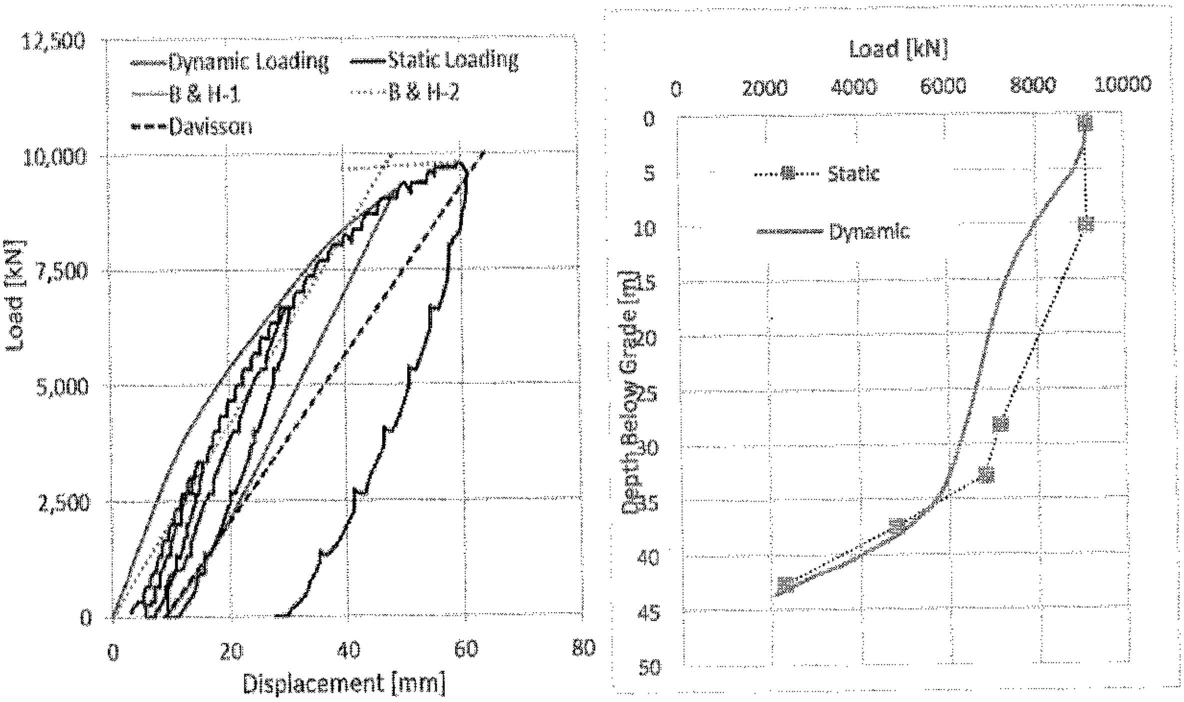

Fig. 13. Left: Load–Displacement curves from static and dynamic loading test with Butler-Hoy and Davisson Criteria indicated; Right: forces measured in the pile at the maximum dynamic loading.

6. SUMMARY

Ideally, the monitoring of pile installation, both by driving and drilling, would avoid the need for additional quality assurance methods and both pile installation recording and dynamic monitoring are a great help in these efforts. However, the complexity of soil behavior and construction procedures require testing of the final deep foundation elements though good construction monitoring may reduce the number of required tests.

A variety of structural (integrity) and geotechnical (loading) tests have been developed, all with their specific benefits and limitations. Dynamic methods, based on wave propagation theory, generally have the advantage of being the least costly and least time consuming.

With proper care and good loading equipment (drop weights), today's dynamic loading tests are of high quality and yield good agreement with static tests. However, differences due exist; they can often be avoided by understanding the effects of the dynamic loading on the soil and potential error sources in both static and dynamic test procedures and interpretations. Increased use of instrumented dynamic and static testing will lead to further understanding of the load transfer in the subsurface materials, promising greater acceptance and therefore improved economies of deep foundations.

Dynamic testing checks both bearing capacity and pile integrity, features which make them widely accepted and specified. Modern building codes have recognized this and included them as an alternative to traditional methods. These codes tend to reward a comprehensive testing program with lower factors of safety and therefore with improved economy (Likins, 2015).

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