# Pile behaviour—theory and application

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This paper discusses the application of theory to the analysis of pile foundation behaviour under axial loading. A classification system is suggested for pile analysis and design procedures, based on the rigour of the underlying theory. It is shown that a number of the analyses in use have a common underlying theoretical basis founded on the boundary element method and that such methods give solutions which are consistent with those from other methods such as the finite element method. The main characteristics of pile behaviour are reviewed for single piles and pile groups subjected to static loading, cyclic loading, and to loading arising from externally-imposed soil movements. Correlations are then summarized for the geotechnical parameters required for the prediction of axial pile behaviour. Using characteristic values of these parameters, design charts are developed for the load capacity and settlement of piles and pile groups. Finally, three case studies are described which demonstrate the sensitivity of pile performance predictions to the method of analysis, the idealization of the soil profile, and the selection of soil parameters. It is shown that the method of analysis is likely to have less effect on the predicted performance than does the geotechnical characterization of the site.

KEYWORDS: analysis; case history; design; piles; repeated loading; settlement.

L'article discute l'application de la théorie à l'analyse du comportement des fondations par pieux sous chargement axial. On propose un système de classification basé sur la rigueur de la théorie associée pour l'analyse des pieux et les procédés de construction. On démontre que quelquesunes des analyses employées ont une base théorique commune dérivant de la méthode des éléments limites et que de telles méthodes donnent des solutions qui s'accordent bien avec celles obtenues à partir d'autres méthodes, telles que les éléments finis. Les caractéristiques principales du comportement des pieux sont examinées pour les pieux et les groupes de pieux soumis aux chargements statiques et cycliques aussi bien qu'au chargement résultant des mouvements du sol imposés de l'extérieur. Puis on résume les corrélations pour les paramètres géotechniques nécessaires pour la prédiction du comportement axial des pieux. À l'aide de valeurs caractéristiques de ces paramètres on propose des abaques de construction pour la force portante et le tassement des pieux et des groupes de pieux. Trois cas réels sont présentés. Ils illustrent la façon dont les prédictions du comportement des pieux dépendent de la méthode d'analyse, de la schématisation du profil du sol et de la sélection des parametres du sol. On démontre que la performance prévue est probablement moins influencée par la méthode d'analyse que par la détermination des caractéristiques géotechniques du site.

# INTRODUCTION

For many years, the design of pile foundations was based on a combination of empiricism and experience, and the general attitude towards theoretical analysis of pile foundations was exemplified by Terzaghi & Peck (1967), who stated

"... theoretical refinements in dealing with pile problems... are completely out of place and can be safely ignored."

Despite this pessimistic evaluation, the past three decades have seen a gradual change in pile design

procedures, from essentially empirical methods, towards methods with a sounder theoretical basis. This change has resulted from a number of stimuli, including the wider use of piling, the recognition that pile foundations do indeed settle and that such settlements must be controlled, and the need to support very large loads on piles, especially for the foundations of offshore structures. In the latter case injudicious extrapolation of previous experience with small onshore piles may be hazardous, particularly as the loads of offshore piles will generally involve a cyclic component, and the soils encountered may exhibit unusual characteristics.

Allied to these stimuli for improved design pro-

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cedures has been the rapid development of powerful numerical techniques, in particular the finite element method and the boundary element method. These, in conjunction with the now commonplace availability of powerful desktop computers, have made feasible methods of pile analysis which even a decade ago would not have been considered practical for foundation design. Several such analysis methods have been proposed and various claims have been made regarding the superiority of one or other of these methods over alternative approaches. It would therefore appear appropriate at this time to review some of these methods, examine similarities and differences among them, and obtain some insight into the significance of such differences when compared with uncertainties which may arise in the practical application of the methods.

This Paper therefore has the following objectives

(a) to present a classification of methods for pile foundation analysis and design

- (b) to outline a versatile analytical framework which can incorporate several existing methods of analysis
- (c) to compare alternative analytical procedures
- (d) to examine the characteristics of behaviour of single piles and pile groups under static axial loading, cyclic axial loading, and when subjected to externally-imposed soil movements
- (e) to review methods of obtaining the soil parameters required for the prediction
- (f) to present examples of simplified design charts based on theoretical analysis and characteristic soil parameters
- (g) to discuss the application of theory to practical cases, and to examine the sensitivity of pile performance predictions to a variety of factors over which the geotechnical analyst has control, including the method of analysis, and the geotechnical characterization of the site.

Attention will be confined to problems involving static or quasi-static axial loading of piles and pile groups in soil. Both load capacity and defor-

Category	Subdivision	Characteristics	Method of parameter determination
1	_	Empirical—not based on soil mechanics principles	Simple in situ or laboratory tests, with correlations
2	2 <b>A</b>	Based on simplified theory or charts—uses soil mechanics principles— amenable to hand calculation. Theory is linear elastic (deformation) or rigid plastic (stability)	Routine relevant in situ tests— may require some correlations
	28	As for 2A, but theory is non-linear (deformation) or elasto-plastic (stability)	
3	3A	Based on theory using site- specific analysis, uses soil mechanics principles. Theory is linear elastic (deformation) or rigid plastic (stability)	Careful laboratory and/or in situ tests which follow the appropriate stress paths
	3B	As for 3A, but non-linearity is allowed for in a relatively simple manner	
	3C	As for 3A, but non-linearity is allowed for by way of proper constitutive models of soil behaviour	

Table 1. Categories of analysis/design procedures

mation are considered, but it is emphasized at the outset that load capacity and deformation are not independent and, although they can be separated with reasonable justification for many problems involving conventional direct loading, their interdependence may be very important for problems involving cyclic loading or external soil movements.

# CATEGORIES OF ANALYSIS AND DESIGN PROCEDURES

Analysis and design procedures can be divided into three broad categories, depending on the level of sophistication and rigour. An extended classification system of these procedures has been proposed by Poulos & Hull (1989) and is shown in Table 1. Category 1 procedures probably account for most pile design done throughout the world. Category 2 procedures have a proper theoretical basis, albeit simplified, and are being increasingly used for pile deflexion calculations. Such procedures involve the use of simple computational methods or design charts, and generally do not demand the use of a computer. Category 3 procedures involve the use of a site-specific analysis based on relatively advanced analytical or numerical techniques such as the finite element method or the boundary element method. In most cases, such procedures require the use of a computer. Category 3 procedures are frequently used to carry out the necessary parametric solutions and develop the design charts which can then be used as category 2 solutions.

Typical examples of the various categories of procedures for axially loaded piles are shown in Table 2. In choosing an appropriate category of design for a practical problem, the following factors need to be considered

- (a) the significance and scale of the problem
- (b) the available budget for foundation design
- (c) the geotechnical data available
- (*d*) the complexity of both the geotechnical profile and the design loading conditions

Category	Axial pile capacity	Settlement
1	Correlations with CPT (e.g. Schmertmann, 1975; De Ruiter & Beringen, 1979). Correlations with SPT (Thorburn & McVicar, 1971); Meyerhof, 1956) Total stress (α) method (Tomlinson, 1957; Semple & Rigden, 1984)	Approximate correlations with pile diameter (Meyerhof, 1959; Frank, 1985) Column deflexion multiplied by a factor (Focht, 1967)
2A	Effective stress (β) method (Burland, 1973; Meyerhof, 1976; Stas & Kulhawy, 1984)	Elastic solutions (Randolph & Wroth, 1978; Poulos & Davis 1980)
28	Effective stress method (Fleming <i>et al.</i> , 1985)	Elastic solutions modified for slip (Poulos & Davis, 1980)
3A	Plasticity solutions for, end bearing capacity (Giroud et al., 1973; Meyerhof, 1963)	Elastic finite element analysis (e.g. Valliappan <i>et</i> <i>al.</i> , 1974)
3 <b>B</b>	Non- linear load transfer analysis (e.g. Coyle Reese, 1966; Kraft <i>et al.</i> , 1981)	e &
	Non-linear boundary element analysis (e.g. I & Davis, 1980) Non-linear finite element analysis (e.g. Desai 1974; Jardine <i>et al.</i> , 1986)	Poulos
3C	Finite element analysis, including simulation pile installation (e.g. Nystrom, 1984; Rand et al., 1979; Withiem & Kulhawy, 1979)	of lolph

Table 2. Examples of categorization of methods for evaluation of axial pile response

- (e) the stage of the design process (i.e. whether a feasibility, preliminary or final design is being carried out)
- (f) the experience of the designer with the methods being considered.

Small projects with a limited budget for geotechnical work rarely justify more than a category 1 or 2A approach. However, for final foundation design in a major project for which considerable geotechnical data has been obtained, a category 3 approach would be appropriate. If, for the same project, a preliminary design was required based only on limited in situ or laboratory data, a category 2 approach might be useful for carrying out sensitivity studies to identify those parameters which are most significant and need to be determined with greatest care.

There will often be occasions in practice where a category 3 analysis is required to account for some unusual aspect of the soil profile or the unconventional nature of the design loading (e.g. if it involves a significant cyclic component). It is not unlikely that the parameters which are required for such a category 3 analysis will have to be estimated rather crudely, e.g. from empirical correlations which would be more appropriate to category 1 or 2 procedures. Consequently, it cannot be expected that solutions from a category 3 analysis will always be superior to those from a simpler analysis. As will be illustrated in this Paper, the method of analysis frequently is of much less importance than the geotechnical parameters which are selected and the way in which the geotechnical profile is idealized.

# ANALYTICAL FRAMEWORK FOR AXIALLY LOADED PILES

In the past two decades, a number of methods have been developed for analysing axially loaded piles, many of which fall into category 3. Most of these methods involve the use of one (or more) of the following analytical techniques

- (a) simplified analytical methods involving the consideration of independent horizontal 'slices' of pile and soil, e.g. Randolph & Wroth (1978).
- (b) boundary element methods, employing either load-transfer functions to represent the interface response (e.g. Coyle & Reese, 1966; Kraft et al., 1981) or elastic continuum theory to represent the soil mass response (e.g. Butterfield & Banerjee, 1971; Banerjee, 1978; Banerjee & Davies, 1978; Poulos & Davis, 1980).
- (c) Finite element methods (e.g. Desai, 1974; Valliappan et al., 1974; Balaam et al., 1975; Ottaviani, 1975; Jardine et al., 1986), in which a

variety of constitutive soil models can be utilized, and such factors as soil nonhomogeneity and anisotrophy can be taken into account.

Finite element methods offer the most powerful analytical approach in that, not only can nonlinear soil behaviour be modelled, but the complete history of the pile can be simulated, i.e. the processes of installation, reconsolidation of the soil following installation, and subsequent loading of the pile (e.g. Nystrom, 1984; Withiam & Kulhawy, 1979). Such analyses are valuable in leading to a better understanding of the details of pile behaviour, but are unlikely to be readily applicable to practical piling problems because of their complexity and the considerable number of geotechnical parameters required.

A reasonable compromise between excessive complexity and unacceptable simplicity is provided by boundary element methods, in which the pile-soil interface is discretized and the characteristics of the soil response are represented in a lumped form by ascribing the behavioural features of the soil to the interface elements. This method has been developed by a number of research workers and is widely used in practice, and attention will therefore be focused on this method. Although a considerable number of formulations exist, most appear to have a common underlying basis. A convenient means of developing a unified analysis is to employ a substructuring technique in which the pile (or piles) and the surrounding soil are considered separately and then compatibility conditions are imposed.

The behaviour of each element is considered at a node which is located at the centre of that element and along a common vertical plane through the pile axis. Fig. 1 shows the division of a single pile into elements, the distribution of freefield soil movements due to some external cause (e.g. swelling or consolidation of the soil mass due to moisture changes or external loading) and the specified distributions of

- (a) the limiting pile-soil stresses,  $f_c$  for compressing loading, and  $f_t$  for tensile loading; for shaft elements, the limiting pile-soil resistance will be termed here the shaft resistance, while for base elements, the term end-bearing resistance will be used
- (b) the local stiffness  $k_s$  of the soil.

At this stage, no assumptions are made regarding the nature of  $f_e$ ,  $f_t$  and  $k_s$ ; these may vary with depth, stress (or displacement) level and with time.

The responses of the pile and soil elements to an increment of axial load  $\Delta P$  are analysed in



Fig. 1. Basic problem of single axially loaded pile: (a) geometry; (b) stresses acting on pile elements; (c) stresses acting on soil elements; (d) distribution of soil stiffness with depth; (e) distribution of free-field soil movement with depth; (f) distribution of limiting pile-soil stress

turn. Details of the equation that describes incremental displacements of the pile are given by Poulos (1979b). For the soil elements, the incremental displacement of the soil may arise from two sources, the pile-soil interaction stresses  $\{\Delta p\}$ , and the free-field soil movements  $\{\Delta S_e\}$ . At any element i, the soil will be in one of three states; non-failure, failure in 'compression', or failure in 'tension'.

In the non-failure state, the incremental displacement of the soil at an element i,  $\Delta \rho_{si}$ , can then be expressed as

$$\Delta \rho_{\rm si} = \sum_{\rm j=1}^{\rm m} \frac{I_{\rm ij}}{k_{\rm sij}} \cdot \Delta p_{\rm j} + \Delta_{S_{\rm ei}} \tag{1}$$

where  $I_{ij}$  is displacement influence factor for element i due to element j,  $k_{sij}$  is soil stiffness for element i due to element j,  $\Delta p_i$  is the increment of pile-soil interaction stress,  $\Delta S_{ei}$  is the increment of external soil movement at element i and *m* is the total number of elements into which pile is divided.

In the 'compressive' failure state, the conditions at element i may be expressed as

$$\Delta p_{\rm i} = f_{\rm ci} - p_{\rm i} \tag{2a}$$

and for the 'tensile' failure state,

$$\Delta p_{\rm i} = f_{\rm ti} - p_{\rm i} \tag{2b}$$

where  $\delta p_i$  is incremental pile-soil stress on element i,  $p_i$  is the existing pile-soil interaction stress on element i before the load increment,  $f_{ti}$  is the limiting pile-soil interaction stress in tension, at element i, and  $f_{ci}$  is the limiting pile-soil interaction stress in compression, at element i.

Equations (1), (2a) and (2b) describe the behaviour of the soil, and may be written for all elements of the system. By equating the expressions for incremental pile and soil displacements, invoking the vertical equilibrium condition, and checking the state of each element during the loading increment, the incremental pile-soil stresses can be determined, and hence the pile displacement at each element can be evaluated. Further details of the numerical procedure are given by Poulos & Davis (1980). By successive application of loading increments, the entire load-displacement relationship for the pile may be determined, including any unloading and reloading sequences.

This analysis becomes less satisfactory as the pile becomes very short or its stiffness relative to the surrounding soil decreases. For pile sockets in rock, the finite element method provides more reliable solutions, e.g. Pells & Turner (1979). The analysis in this section can be used for both the load-transfer analysis (commonly termed the t-z analysis), in which the soil resistance is modelled by a discrete spring at each element, and the continuum analysis, in which the soil is modelled as an elastic continuum.

For the load-transfer or t-z analysis, the displacement influence factor  $I_{ij}$  in equation (1) is zero, except when i = j, in which case  $I_{ii} = 1$ ; in other words, the soil deflexion of a node i depends only on the stress at that node. The soil stiffness in equation (2) can be evaluated either empirically (e.g. Coyle & Reese, 1966) or theoretically (Baguelin & Frank, 1980; Kraft *et al.*, 1981;

Ha & O'Neill, 1983; Chow, 1986; Ooi et al., 1989). Such features as soil non-homogeneity, non-linearity, post-peak softening, and interface dilation or contraction, may be incorporated. The main limitation of the load-transfer analysis is that it can only properly consider a single pile, and cannot be used directly to analyse pile groups. It is possible to modify the single pile t-zcurves to allow for group effects (Randolph, 1986) but it is usually necessary to evaluate these group effects using elastic continuum theory (e.g. Chow, 1986).

In the continuum analysis the displacement influence factor  $I_{ij}$  (in equation (1)) for all elements is non-zero, and is usually evaluated from elastic continuum theory. The theory of Mindlin (1936) is frequently used for this evaluation (e.g. Poulos & Davis, 1968), although other basic elastic solutions may be employed, for example, the equations of Chan *et al.* (1974) for a layered half-space, as utilized by Chin (1988).

An implicit assumption in the elastic continuum analysis is that, if incremental tensile stresses are developed in the soil mass by the loading of the pile, the soil continues to respond elastically. Such an assumption is reasonable if the overall stress conditions remain compressive (due to the compressive overburden stresses), but may be questionable if the overall stresses become tensile. Unease about this point has led to some criticism of elastic continuum theory (e.g. Leonards & Darrag, 1989) and a preference for the loadtransfer approach, as the latter involves no consideration of the stress state in the soil other than at the pile-soil interface. However, load-transfer and continuum solutions for a single pile generally agree well and suggest that any effects of possible tension in the soil are unlikely to influence pile behaviour significantly.

# IMPLEMENTATION OF THE THEORY

In this Paper, attention will be concentrated on the continuum analysis. With this approach, it is only possible to derive proper results for quite idealized problems that involve linear soil behaviour and one- or two-layer elastic soil masses. The range of problems for which proper results can be derived can be extended by using the finite element method to evaluate the soil displacement influence factors. However, it is also possible to utilize continuum theory in an approximate manner to obtain solutions for a much wider range of problems of practical interest.

Figure 2 gives an indication of the extensions to the theory which may be accomplished by using the continuum analysis. The extensions are classified into six groups, two of which involve the modelling of the soil, two of which involve the modelling of the piles, and the last two which involve the modelling of the loading on the piles.

### Modelling the soil profile

It is convenient to use the theory of elasticity for modelling the soil behaviour. Despite the gross simplification which this model involves when applied to real soil, it provides a useful basis for the prediction of pile behaviour, provided that appropriate equivalent elastic parameters are selected for the soil. A significant advantage of using an elastic model for soil is that it provides a rational means of analysis of pile groups and evaluation of immediate and final movements of a pile. In determining immediate movements, the undrained elastic parameters of the soil are used in the theory, whereas for final movements the drained parameters are used.

Mindlin's equations may be used to obtain approximate solutions for a layer of finite thick-



Fig. 2. Basic analysis, and extensions to allow for more realistic modelling

ness by using the Steinbrenner approximation (Steinbrenner, 1934) to allow for the effect of the underlying rigid base in reducing the soil displacements. For piles bearing directly on a stiffer stratum, the reduction in soil displacement may be analysed approximately by using the mirrorimage technique described in Poulos & Davis (1980). In both these cases, the soil displacement factors  $I_{ii}$  in equation (1) are modified.

In the elastic continuum theory, the soil stiffness  $k_{ij}$  in equation (1) is directly related to the Young's modulus of the soil. In a homogeneous soil mass,  $k_{sij} = E_s/d_j$  where  $E_s$  is the (constant) Young's modulus and  $d_j$  is the diameter of element j. If a non-homogeneous soil mass is considered and Mindlin's equation is used to evaluate the displacement influence factors, an approximation is required to determine  $k_{sij}$ . Poulos (1979a) has examined alternative assumptions and concluded that the following approximation is generally satisfactory

$$k_{\rm sii} = 0.5(E_{\rm si} + E_{\rm si})/d_{\rm i} \tag{3}$$

where  $E_{si}$  and  $E_{sj}$  are the values of soil Young's modulus at elements i and j respectively.

Equation (3) becomes inaccurate if large differences in soil modulus exist between adjacent elements or if a soil layer is overlain by a much stiffer layer. In a subsequent investigation, Yamashita *et al.* (1987) have developed a more general approximation in which  $E_{si}$  and  $E_{sj}$  in equation (3) are taken as weighted average values of soil modulus, involving the value of soil modulus at all elements along and beneath the pile.

### Modelling of soil behaviour

One of the more important features of pile-soil behaviour is the limited stress which can be developed between the soil and the pile. This feature is already incorporated into the analysis in that checks are made to ensure that the pile-soil shear stress does not exceed the specified limiting value. If the interface behaviour at an element is assumed to be perfectly linear until the limiting stress is reached, the interface model will be termed (conveniently, but not strictly correctly\*) an 'elastic-plastic' interface model.

It is possible to incorporate non-linear behaviour of the soil, in an approximate manner, by assuming that the soil Young's modulus varies with either stress or strain level. The simplest assumption is of a hyperbolic relationship between soil shear stress and shear strain, in which case the tangent Young's modulus of the soil  $E_{tan}$  is given by

$$E_{\rm tan} = E_{\rm t} \left( 1 - \frac{pR_{\rm f}}{p_{\rm f}} \right)^2 \tag{4}$$

where  $E_t$  is the initial tangent Young's modulus,  $R_f$  is the hyperbolic curve-fitting constant, p is the pile-soil stress,  $p_f$  is the limiting value of pile-soil stress ( $p_f = f_c$  in compression, or  $f_t$  in tension).

It is also possible to use alternative models to describe non-linear behaviour, for example, multilinear models, or the Ramberg-Osgood model. Regardless of which non-linear model is used, it is important to consider unloading and reloading, as well as initial loading. The simplest assumption is to assume that on unloading or reloading, the behaviour is elastic until the previous greatest stress level at that element is reached; thereafter, non-linear initial loading resumes. However, alternative assumptions are possible in which non-linear behaviour recommences at stress levels less than the previous greatest level (Randolph, 1986). Such behaviour can give rise to the accumulation of permanent displacements if repeated loading is applied, but additional parameters are then required in the analysis.

In analysing the behaviour of piles subjected to cyclic loading, at least three aspects of soil response should be considered,

- (a) the degradation of pile-soil resistances (and possibly soil modulus) under repeated cyclic loading
- (b) loading rate effects
- (c) the accumulation of permanent displacements.

One possible approach to incorporating cyclic loading effects has been described elsewhere by the Author (Poulos, 1988b). The effects of cyclic degradation are most significant for shaft resistance, and may be conveniently quantified by means of a degradation factor  $D_f$ , where

$$D_{\rm f} = \frac{\text{element after cyclic loading}}{\text{shaft resistance at that}}$$
(5)  
element for static loading

The model of Matlock & Foo (1980) is useful for quantifying the change in  $D_f$  as cycling proceeds, and involves only two parameters, a minimum degradation factor  $D_{lim}$  and a degradation rate parameter  $\lambda$ .

The effects of loading rate are incorporating by means of a loading rate factor  $D_R$ , while permanent displacements developed in the soil during cyclic loading are considered to be equivalent to cyclic loading-dependent external soil

<sup>\*</sup> Such a model does not necessarily imply that the relationship between interface stress and displacement at an element is elastic-plastic; because of the continuum nature of the soil, the slope of this relationship will change as other interface elements reach a failure state.

movements. Other aspects of soil behaviour may also be included, for example, strain-softening interface behaviour (Randolph, 1983; Kraft *et al.*, 1981) and creep (Booker & Poulos, 1976).

# Pile characteristics

The analysis can readily accommodate piles of varying diameter or varying stiffness within the formulation of the pile displacement equation. In the case of varying diameter, additional annular elements are introduced at the diameter discontinuities, and values of the limiting pile-soil resistance at these elements, for both compression and tension loading, must be specified.

It is also possible to allow for failure of the pile material itself (either in compression or tension) by limiting the axial stress that can be developed in the pile section. The procedures for making this modification are described in Poulos & Davis (1980).

# Pile groups

The continuum-based analysis can readily be extended to analyse a group of axially loaded piles. Each pile is discretized into elements, soil and pile displacement equations are assembled for each element, compatibility of soil and pile displacements is imposed at elements in the nonfailure state, and the vertical equilibrium equation for each pile is written. In addition, the pile head conditions must be specified. Usually, a rigid cap connects the piles, so that all piles will undergo an equal head displacement. By specifying this condition, a further set of equations is obtained, which enables the load increment on each pile head to be computed, in addition to the distribution of incremental pile-soil stress and displacement (Hewitt, 1988).

In the evaluation of the displacement influence factors  $I_{ij}$ , use may again be made of the elastic solution of Mindlin (1936) to determine the displacement of an element of a pile due to all elements of that pile and the other piles in the group. A convenient approximation for this evaluation has been developed by El-Sharnouby and Novak (1985) which avoids the need for double integration of the Mindlin equation and hence reduces the computation time substantially. A number of methods of implementing the above analysis have been developed, including the following.

(a) A complete analysis of the group is performed using a continuum analysis—this has been done by Banerjee & Driscoll (1976) and Poulos & Hewitt (1986).

- (b) A complete analysis of the group can be performed, using a load-transfer analysis to determine the response of a pile to its own load, and continuum theory to determine the influence of the elements of the other piles this has been termed the hybrid method and has been used by O'Neill et al. (1977) and Chow (1986).
- (c) Solutions for a two-pile group (obtained from a continuum analysis) can be used to obtain interaction factors that express the increase in head settlement of a pile due to the presence of another pile (Poulos, 1968; Randolph & Wroth, 1979). The interaction factor  $\alpha$  is defined as

$$\alpha = \Delta S / S_1 \tag{6}$$

where  $\Delta S$  is the increase in settlement of a pile due to the presence of another equally loaded pile and  $S_1$  is the settlement of a single pile under its own load. For a group of piles, the interaction factors may be superposed to develop a set of equations relating the settlement of each pile to the single pile settlement, the load on each pile head, and the interaction factors.

(d) A modified form of the interaction factor method can be employed in which different values of soil Young's modulus are used to determine the single pile behaviour and the interaction factors. Higher values are generally used for the latter to reflect the lower level of strain (and hence the greater stiffness) of the soil between the piles as compared with the soil immediately adjacent to each pile. This method is termed the modified interaction factor method, and has been described in detail by Poulos (1988a).

Comparisons between these approaches will be discussed later in the Paper.

# Loading conditions

**Residual stress effects.** Most analyses of pile response assume an initially stress-free pile, although it is well-recognized that residual stresses exist in piles, due to installation effects, particularly in driven piles. Ideally, the driving of a pile should be modelled using a dynamic analysis such as that used by Holloway *et al.* (1978). However, a simpler first approximation can be employed in which the pile (at final penetration) is loaded to failure in compression and then unloaded back to zero load. Poulos (1987) shows that this procedure gives residual stresses in the pile which appear to be reasonably realistic. Leonards & Darrag (1989) point out that assumptions made regarding the soil modulus near the pile tip can significantly influence the computed residual stresses in the lower part of the pile, particularly for piles in sand.

**Cyclic loading.** At least two methods can be adopted to simulate cyclic loading of a pile

- (a) a single-step analysis, in which a single iterative analysis is carried out to determine the behaviour of a pile after a specified number of cycles, N
- (b) a cycle-by-cycle analysis, in which the application of each of the N cycles is modelled in turn.

Details of these methods are given by Poulos (1983). The cycle-by-cycle analysis is considerably more time-consuming, but much more versatile, in that it can accommodate strain-softening interface behaviour and sequences of irregular cyclic loading.

External soil movements. The effects of external soil movements, such as those arising from soil consolidation due to external loading or dewatering, or from soil heave due to wetting of expansive clay layers, can be analysed directly, provided that the external free-field soil movements at each element (i.e. values of  $\Delta S_{ei}$  in equation (1)) can be specified. In the case of one-dimensional loading of a clay layer, these movements may be determined from Terzaghi's one-dimensional consolidation theory, and the time-dependency of the pile behaviour can readily be determined (Poulos & Davis, 1980). The freefield movements in an expansive soil layer are less easily computed, although a number of empirical or semi-empirical methods exist (e.g. Blight, 1965; Van Der Merwe, 1964; Cameron & Walsh, 1984).

#### Computer codes

The preceding analyses almost invariably require computer evaluation, and a number of computer programs have been written for examining various aspects of axial pile behaviour. A selection of these is given in Table 3. This list is by no means exhaustive and there is no doubt a great number of other codes in existence. Also excluded from this list are codes based on finite element analysis.

# COMPARISONS BETWEEN ALTERNATIVE THEORIES

The preceding section has discussed alternative approaches to the analysis of pile behaviour, within a single framework of the boundary element method. It is of interest to compare these alternative approaches to determine the sensitivity of the computed behaviour to the approach adopted. At the same time, it is of considerable interest to compare the solutions with an independent analysis method such as the finite element method. Therefore, this section will make such comparisons for three different problems: (a) a single axially loaded pile; (b) an axially loaded pile group; and (c) a pile subjected to downdrag forces by external vertical soil movements.

#### Single pile

For a single pile having a typical relative compressibility  $K = E_p/E_s$  of 1000, (where  $E_p$  is the Young's modulus of pile and  $E_s$  is Young's modulus of soil) in a deep homogeneous elastic soil mass, Fig. 3 compares two sets of solutions for the settlement as a function of the length-todiameter ratio L/d, the boundary element approximate analytical approach, and the approach developed by Randolph & Wroth (1978). For L/d > 15, the agreement is close, the difference being less than 10%. For smaller values of L/d, the Randolph & Wroth solution gives smaller settlements, possibly due to the effects of the assumption of the soil surrounding the pile being a series of concentric cylinders. A similar measure of agreement is found between the Author's solution and the finite element solutions of Valliappan et al. (1974).

Figure 4 shows various solutions for the settlement of a single pile in a Gibson soil layer. There is reasonable agreement among the three solutions compared but as in Fig. 3, the greatest potential for differences appears to be for relatively short piles (L/d < 15).

In comparing non-linear solutions for single pile response, the problem considered is that analysed by Jardine *et al.* (1986). They have employed a finite element analysis involving the use of a non-linear soil model, the LPC2 model,



Fig. 3. Comparison between solutions for settlement of single pile in deep uniform elastic soil layer

Problem addressed	Program name	Reference	Remarks
Settlement of single pile	TAPILE	Poulos (1978)	t–z analysis continuum analysis
Settlement of pile group (can also be	DEFPIG	Poulos (1980)	Non-linear continuum analysis, using interaction factors
single piles)	GAPFIX	Hewitt (1988)	Non-linear continuum analysis, complete solution
	PIGLET	Randolph (1987)	Simplified continuum analysis, using interaction factors
	PGROUP	Banerjee & Driscoll (1976)	complete linear continuum analysis
	PILGPI	O'Neill et al. (1977)	Non-linear hybrid analysis
	—	Chow (1986)	Continuum-based non-linear hybrid analysis
Cyclic loading	RATZ	Randolph (1986)	Cycle-by-cycle t-z analysis
	AXCYC	Poulos (1988b)	Continuum based cycle-by-cycle analysis
	GAPCYC	Hewitt (1988)	Continuum-based cycle-by-cycle analysis for groups
External soil movements	PNEGA	Kuwabara & Poulos (1989)	Continuum-based analysis for downdrag on end- bearing piles
:	PIES	Poulos (1989)	Continuum-based analysis for pile in shrinking or swelling soil

Table 3. Some computer codes for pile analysis

in which the Young's modulus decreases markedly as the axial strain level increases. The pile is 30 m long, 0.75 m in dia. and is located in a homogeneous soil layer 50 m deep. The initial tangent modulus of the soil (for very low strains) is 1056  $MN/m^2$ , Poisson's ratio is taken as 0.49, and a constant shaft resistance of 220  $kN/m^2$  is assumed. Two values of pile Young's modulus have been considered, 30 000  $MN/m^2$  and 30 000  $GN/m^2$  (the latter would be unrealistically stiff in practice).

Analyses have been performed using the following three boundary element analyses

- (a) an elastic-plastic continuum-based interface model, using a constant soil Young's modulus of 1056 MN/m<sup>2</sup>
- (b) a hyperbolic continuum-based interface model, using an initial tangent soil Young's modulus of 1056 MN/m<sup>2</sup>, a constant shaft resistance of 220 kN/m<sup>2</sup>, and a hyperbolic curve fitting constant  $R_f$  of 0.9 for both the shaft and the pile tip
- (c) a load transfer analysis in which the interface response at each element is elastic-plastic, the linear portion being derived from the initial tangent soil modulus of 1056 MN/m<sup>2</sup>.



Fig. 4. Comparison between solutions for settlement of single pile in Gibson soil layer of finite depth

Figure 5 shows the comparisons between the four theoretical load-settlement curves. The following observations may be made

- (a) for the more compressible (and realistic) pile, all four analyses correspond quite closely; indeed, the finite element analysis and the two elasto-plastic analyses agree remarkably well
- (b) for the stiffer pile, the agreement between the computed load-settlement curves is not as close; at a load of one-half of the ultimate, the predicted settlements from three of the analyses are within  $\pm 10\%$ , but as failure is approached, the curves diverge, and the hyperbolic model in particular predicts larger settlements than the other three methods.

It is clear that, for very stiff piles, the details of the pile-soil interface model have a greater influence on the load-settlement behaviour than for more compressible piles. It is, however, encouraging to observe that the simpler category 3B boundary element analyses are capable of predicting a very similar load-settlement response to that from a category 3C non-linear finite element analysis.

### Pile groups

For a  $3 \times 3$  group in a homogeneous elastic soil layer, Table 4 shows solutions from four different simplified approaches using computer programs listed in Table 3. There is generally very good agreement among the various solutions, for both very compressible and very rigid piles in a deep layer and in a relatively shallow layer. An exception is the PIGLET analysis for the very compressible pile group, in which case the predicted settlement is significantly greater than the other three methods. This difference appears to be associated with the approximations used in PIGLET for the interaction factors; when these same approximations are used in the DEFPIG analysis for the very compressible piles, similarly large settlements are obtained. However, for stiffer piles, these approximations appear to be quite adequate.

Figure 6 shows solutions for the settlement of groups in a Gibson soil. For a finite soil layer (Fig. 6a), the full boundary element solutions of Banerjee & Davies (1977) are in fair agreement with the DEFPIG solutions, although the latter settlements are larger. For an infinitely deep layer, Fig. 6b shows good agreement between the DEFPIG and PIGLET solutions.

Comparisons between boundary element and finite element methods have been made by Poulos (1976) and Pressley & Poulos (1986). The accuracy of some of the finite element solutions is difficult to assess, but in general they agree reasonably well with the boundary element solutions. Cheung *et al.* (1988) have developed an infinite layer method to obtain two-pile interaction factors for use in a pile group analysis. The

Table 4. Comparisons between solutions for group settlement in homogeneous soil: L/d = 40,  $v_s = 0.49$ ,  $3 \times 3$  group, s/d = 3.0;  $S = (P_G/dE_s)I_G$ 

Method	Values of I <sub>G</sub>						
	h/L = 1.67		$h/L = \infty$				
	<i>K</i> = 30	K = 30000	K = 30	K = 30000			
DEFPIG GAPFIX PIGLET Butterfield & Douglas (1981)	0.063 0.060  0.058	0.021 0.021  0.020	0.069 0.069 0.105 0.067	0.029 0.029 0.026 0.025			



interaction factors thus obtained are in close agreement with the values of Poulos & Davis (1980) determined from the boundary element analysis.

O'Neill & Ha (1982) have compared the behaviour of pile groups predicted by the program DEFPIG with that predicted by a kind of hybrid analysis implemented by means of the program PILGPI. It is found that the two programs give comparable predicted behaviour, although different values of soil Young's modulus are required to give exact numerical agreement. These comparisons reinforce the fact that the value of soil modulus is not unique but must be selected carefully for use with the method of analysis employed.

In summary, while the foregoing comparisons demonstrate some differences among various methods, they also indicate generally satisfactory agreement between the boundary element solutions using interaction factors and solutions from the other approaches. Finite element analyses can be illuminating in that they reveal detailed behavioural characteristics, but it would appear that adequate practical predictions of group settlement can be obtained from simpler approaches based on boundary element analysis.

### Single pile subjected to vertical soil movements

Small (1988) has used the finite element method to analyse a single end-bearing pile resting on a rigid base, in a soil layer which is subjected to external loading so that vertical soil movements occur and downdrag (or negative friction) forces are developed in the pile. The problem is illustrated in Fig. 7 which shows the finite element solution obtained for the development of pile head displacement with time as the soil layer consolidates. Also shown is the solution obtained by Poulos & Davis (1980) from a boundary element analysis, and it can be seen that, for a value of drained Poisson's ratio of the soil of 0.3, the agreement between the two solutions is close. There are, however, some differences in the detail of the developed shear stresses along the pile shaft, especially near the soil surface, possibly because of discretization inaccuracies in both analyses. Other independent analyses of the same general problem by means of continuum-based boundary element analyses (e.g. Chin, 1988; Kog *et al.*, 1986) are in general agreement with the Poulos & Davis solutions.

### CHARACTERISTICS OF PILE BEHAVIOUR

In this section, some of the more significant characteristics of pile behaviour will be itemized. Most have been derived from theoretical analyses but in many cases are also supported by measurements made from laboratory and field tests. Unless otherwise stated, the solutions described will have been obtained from boundary element analyses based on elastic continuum theory, with an elastic, or an elastic-plastic interface model, using simplified distributions of soil Young's modulus and shaft resistance with depth (either constant or linearly increasing).

Four aspects of behaviour will be considered

- (a) the load-settlement behaviour of a single pile under static axial loading
- (b) the settlement of pile groups under static axial loading
- (c) piles subjected to external soil movements
- (d) piles subjected to cyclic axial loading.



Fig. 7. Vertical deflexion of pile head with time



Fig. 8. Influence of dimensionless parameters L/d and K on settlement. Single friction pile in homogeneous soil

Illustrations of the points will be made frequently with reference to hypothetical problems involving realistic soil and pile parameters so that the practical implications of these points may be more readily appreciated.

#### Single pile under static loading

The settlement of a single pile is governed largely by the following dimensionless parameters

- (a) the length-to-diameter ratio L/d
- (b) the pile stiffness factor K, the ratio of the Young's modulus of the equivalent solid pile section  $E_p$  to the Y ag's modulus of the soil  $E_s$

(c)  $E_b/E_s$ , the ratio of the Young's modulus of the bearing stratum at the pile tip to the Young's modulus of the soil.

For the case of a friction (or floating) pile in a homogeneous elastic soil, Fig. 8 shows that the settlement decreases as L/d and K increase. Experimental evidence from model tests in clay presented by Butterfield & Ghosh, 1977) demonstrates that the theory can give a realistic prediction of the effects of L/d.

The settlement of a pile is not significantly influenced by the nature of the bearing stratum if the pile is relatively slender and/or compressible. Fig. 9 compares the settlement of an end-bearing pile relative to a corresponding friction pile, for a typical value of K of 1000. For values of L/d in excess of about 50, the reduction in settlement due to the bearing stratum is less than 40%, even if the bearing stratum is very much stiffer than the overlying soil. Thus, if a reduction in settlement of a long pile is sought, there appears to be little to be gained by founding the pile tip on a stiffer underlying stratum. Increasing the diameter and/or the stiffness of the pile is likely to be more productive. It should also be noted that there is a critical length for a pile, beyond which further increase in length produces no further reduction in settlement. For a friction pile in a homogeneous soil, this critical length is given by the approximate expression (Hull, 1987)

$$L_{\rm c}/d = \left(\frac{\pi E_{\rm p} A_{\rm p}}{E_{\rm s} d^2}\right)^{0.5} \tag{7}$$

where  $A_{\rm p}$  is the area of pile cross-section.



Fig. 9. Relative settlement of end bearing and floating pile. Homogeneous soil



Fig. 10. Influence of distribution of soil Young's modulus on load transfer: (a) stress distribution along shaft; (b) distribution of load

The settlement and load transfer are influenced by the distribution of soil Young's modulus along the pile shaft. An example is given in Fig. 10 for a typical friction pile in two different soils: a homogeneous soil, and a Gibson soil in which the soil modulus increases linearly with depth, from zero at the soil surface. For the homogeneous soil, the distribution of shear stress  $\tau$  is relatively uniform with depth, whereas for the Gibson soil,  $\tau$ increases with depth. The similarity between the stress distribution and the distribution of soil Young's modulus may explain why load transfer approaches can give reasonable predictions of pile behaviour. For the same average value of Young's modulus along the pile shaft, the pile head settlements in this case are reasonably similar, and differ by only about 7%. However, the tip settlement for the pile in the Gibson soil is about 18% less than that of the pile in the homogeneous soil.

The major part of the final settlement of a single pile is immediate settlement, and occurs on application of the load because the load is transferred to the soil essentially by shear, with relatively little change in mean stress. For practical values of  $v_s'$  (of the order of 0.3–0.4) and L/d, the theory suggests that the ratio  $S_i/S_{TF}$  of immediate to final settlement of a pile in a homogeneous soil

is in excess of 0.85, and is almost independent of L/d. This theoretical conclusion is supported by the results of model tests on brass piles in kaolin, reported by Mattes & Poulos (1971) and field maintained loading tests, e.g. Whitaker & Cooke, 1966. A corollary to the above observations is that the *rate* of consolidation settlement of piles in clay is not likely to be an important consideration in design. Time effects stemming from creep at higher load levels are likely to be more important than consolidation time effects (Edil & Mochtar, 1988).

At normal working loads (of the order of 40–50% of the ultimate load), non-linear behaviour of the soil generally does not have a substantial influence on pile settlement. Careful model tests reported by Butterfield & Abdrabbo (1983) support this theoretical conclusion.

For a typical pile in a homogeneous soil, Fig. 11 gives some indication of the potential influence of non-linear soil behaviour on settlement. The ratio of settlement of the pile in a purely elastic soil  $S_{elas}$  to the settlement of the pile in a soil with hyperbolic response S, is plotted as a function of the load level  $P/P_u$ , where  $P_u$  is the ultimate load capacity. The initial tangent Young's modulus of the hyperbolic soil model is assumed equal to the modulus of the purely elastic soil.  $S_{elas}/S$  is gener-



Fig. 11. Example of influence of nonlinearity on computed pile settlement. Hyperbolic interface model

ally less than unity (indicating that non-linearity results in an increase in the settlement), the extent of this increase depending on the hyperbolic parameters adopted. For the parameters considered to be most realistic (curves c and d), nonlinearity causes an increase of between 11 and 25% in the settlement, as compared with the settlement determined from a purely elastic analysis using the initial tangent Young's modulus of the soil. It should be noted that, in this case, the nonlinearity arising from pile-soil slip is only significant at load levels close to failure and therefore that the use of elastic theory with an appropriately reduced secant modulus would give a reasonable prediction of settlement. However, for slender compressible piles, pile-soil slip may have a more dominant influence on the non-linearity of the load-settlement response.

Residual stresses that remain in the pile after installation may influence the pile head stiffness and hence the calculated pile head movements. The theoretical analysis of Poulos (1987) has demonstrated that, for cases in which significant residual stresses remain (e.g. for a driven pile in dense sand), the stiffness of the pile head in tension may be smaller than in compression. Fig. 12 illustrates the point for an elastic-plastic interface. If no account is taken of residual stresses, the pile head stiffness in tension and compression is the same, but if residual stresses are allowed for, substantially larger movements can occur in tension than in compression. Under zero net load, the residual stresses are such that tensile pile-soil slip occurs over a significant amount of the pile shaft, and a residual compressive load is developed at the tip. Resistance to applied tensile loading comes primarily from the pile tip, the response of which is less stiff than the pile shaft; therefore, the pile head movements will be greater than if compression is applied.

If the pile-soil interface can strain-soften, both the load-settlement behaviour and the ultimate load capacity will be affected. The ultimate load is no longer statically determinate, but will depend on the relative stiffness of the pile, the ratio of peak to ultimate resistance, and the post-peak behaviour. Randolph (1983) presents solutions for a reduction factor to be applied to pile shaft capacities based on peak values of shaft resistance, in order to allow for the effects of progressive failure along the pile due to strain-softening.

No initial residual stresses



Fig. 12. Effect of residual stresses on load-deflection behaviour pile in sand (Poulos, 1987): (a) compression; (b) tension

These solutions reveal that reductions in the peak load capacity are only likely to be significant for relatively long compressible piles.

### Pile aroup under static loading

Before the development of modern analytical techniques, it was commonly believed that no rational relationship existed between the behavjours of single piles and pile group. In his James Forrest Lecture, Terzaghi (1939) stated

Both theoretical considerations and experience leave no doubt that there is no relation whatever between the settlement of an individual pile at a given load and that of a large group of piles having the same load per pile.

encourage Such statements auite properly caution in dealing with pile groups that contain very large numbers of piles; these are often better considered as a large block foundation. However, for groups that contain relatively few piles, it is possible to link theoretically the settlements of single piles and pile groups. For such groups, under normal working loads, it is convenient to characterize the influence of interaction between piles on the settlement in terms of two dimensionless quantities

- (a) for two piles, the interaction factor  $\alpha$ , which is defined in equation (6) and expresses the relative increase in settlement due to the presence of another pile
- (b) for general pile groups, the group settlement ratio  $R_{\rm c}$ , defined as

$$R_{\rm s} = \frac{\text{settlement of group}}{\text{settlement of single pile}}$$
(8)  
at the same average load

An alternative quantity to  $R_s$  is the group reduction factor  $R_G$ , also termed the efficiency factor by Butterfield & Douglas (1981) and Fleming et al. (1985). R<sub>G</sub> can be defined as

$$R_{\rm G} = \frac{\text{stiffness of group}}{\text{sum of individual pile stiffness}}$$
(9)

For a group of *n* piles,  $R_s$  and  $R_G$  are related as

$$R_{\rm s} = nR_{\rm G} \tag{10}$$

A comprehensive review of the load capacity and settlement of pile groups has been made by O'Neill (1983). Here, attention will be focused primarily on group settlement behaviour, as determined from analyses based on elastic continuum theory. Some of the more significant aspects of behaviour are discussed in the following.

Under working load conditions, pile group interaction depends largely on two sets of dimensionless parameters: those related to the soil and

pile characteristics, and those related to the geometry of the piles and the pile group. The important soil and pile characteristics are the pile stiffness factor K, the ratio of  $E_{\rm b}/E_{\rm s}$  of Young's moduli of bearing stratum to soil, and the distribution of the soil Young's modulus  $E_s$  with depth. Fig. 13 illustrates the influence of these factors on the two-pile interaction factor  $\alpha$ . It may be seen that  $\alpha$  decreases as K decreases, or as  $E_{\rm r}/E_{\rm r}$  increases, or as the distribution of soil Young's modulus becomes less uniform with depth. Consequently, it should be expected that early published solutions for interaction factors, which are for a rigid pile  $(K = \infty)$  in a homogeneous mass  $(E_{\rm b}/E_{\rm s}=1$  and  $E_{\rm s}$  constant with







Fig. 13. Effect of soil-pile parameters on interaction factors: (a) pile stiffness factor K; (b) stiffness of bearing stratum; (c) soil modulus distribution (O'Neill, 1983)

depth), will overestimate the settlement interaction between piles in more realistic situations. The field measurements shown in Fig. 13(c) (O'Neill, 1983) support this contention; however, the theoretical interaction factors for a Gibson soil agree quite well with the measurements.

The primary geometric factors that influence group settlement interaction are the length-todiameter ratio L/d, the relative spacing between the piles s/d, and the number of piles in the group. The effects of s/d are apparent from Fig. 13. For a value of s/d of 4 and K = 1000, the variation of settlement ratio  $R_s$  with L/d and n is shown in Fig. 14 for square groups of friction piles in a Gibson soil of finite thickness h = 2L.  $R_s$ increases as both L/d and n increase. The influence of L/d is small for L/d values in excess of 25. Fleming *et al.* (1985) have presented results for larger numbers of piles which suggest that  $R_s$  can be approximated as follows,

$$R_{\rm s} \simeq n^{\omega} \tag{11}$$

where *n* is the number of piles and  $\omega$  is an exponent which lies between 0.4 and 0.6 for most pile groups. This expression gives results which are reasonably consistent with those in Fig. 14. For a group with a rigid cap, and a given number of piles at a given centre-to-centre spacing, the settlement ratio does not depend to any significant extent on the precise geometrical configuration of the piles, e.g. for a group of 16 piles,  $R_s$  for a



Fig. 14. Influence of geometric parameters on group settlement ratio



Fig. 15. Settlement against group breadth

 $4 \times 4$  configuration is very similar to that for an  $8 \times 2$  configuration.

For a given set of pile characteristics, the group reduction factor (or settlement efficiency factor)  $R_G$  depends largely on the breadth of the group. For groups that contain more than about nine piles, there is an almost unique relationship between  $R_G$  and group breadth. Fig. 15 shows a typical plot, together with data used by Skempton (1953) to derive an empirical design curve. Both the trend and magnitude of the theoretical curves agree well with these data.

In a pile group with a rigid cap, the distribution of load among the piles is generally nonuniform. In a square or rectangular group, the corner piles carry the greatest proportion of load, while those near the centre carry least. Poulos & Davis (1980), O'Neill *et al.* (1982) and Chow (1986) show that the theoretical trends are supported by field and model test data.

Interaction among piles in a group may be influenced by the stiffness of the soil between the piles. Most of the published theoretical solutions assume a soil to be horizontally homogeneous, with the soil Young's modulus between the piles the same as the value adjacent to each pile. However, in reality, the soil between the piles undergoes smaller strains and is likely to be stiffer than near the pile-soil interface, and interaction between the piles will be therefore reduced. A simplified analysis of this effect has been made by Poulos (1988a). For groups of piles, the presence of stiffer soil between the piles leads to a smaller settlement ratio  $R_s$  and a more uniform distribution among the piles, than is predicted by the conventional analysis.

The relative proportion of immediate settlement  $S_i$  to final settlement  $S_{TF}$  decreases as the size of the group increases. For groups of piles in either a homogeneous soil, with a typical value of  $v_s'$  of 0.3, the ratio  $S_i/S_{TF}$  decreases from 0.93 for a single pile to 0.85 for a 25-pile group. Similar values are found for groups in a Gibson soil. The pile stiffness factor K has little influence on  $S_i/S_{TF}$ . For pile groups, the consolidation settlement (and hence the *rate* of consolidation settlement) is more important than for a single pile, but it is still likely to be the minor component of settlement unless soft compressible layers exist beneath the pile tips.

The stiffness of underlying soil layers may have a significant influence on pile group interaction and settlement. Fig. 16 shows an example of the influence of the relative stiffness of the underlying soil on the settlement of a pile group. Clearly, the presence of a softer layer  $(E_b/E_s < 1)$  may substantially increase the settlement, as compared with the case of a homogeneous soil mass  $(E_b/E_s = 1)$ . Also shown is the settlement computed from the approximate approach described by Poulos & Mattes (1971), in which the group is replaced by an equivalent block. This solution agrees well with the solution computed from the interaction factor method.

The effect on group settlement of the pile cap being in contact with the soil is relatively small unless the pile spacing is large and the group is relatively small. Even for piles at an unusually large centre-to-centre spacing of 10 diameters, the reduction in settlement due to cap contact is only about 5%. Therefore, for most practical purposes, the influence of pile cap contact on settlement at working loads can be ignored.

# Piles subjected to external soil movements

There are several circumstances under which loading may be induced in piles by external soil movements, but attention will be confined here to two problems

- (a) end bearing piles subjected to negative friction by settlement of the surrounding soil; the soil movement is assumed to occur over the entire depth of the soil layer, and to vary linearly with depth
- (b) floating piles in a relatively stiff expansive clay, subjected to swelling or shrinking move-



Fig. 16. Effect of modulus of underlying stratum on group settlement

ments; these movements are generally assumed to vary linearly with depth, but to extend to only a limited distance below the surface.

# Negative friction on end-bearing piles

Solutions from purely elastic theory show that the movement and downdrag force induced in the pile depend on the dimensionless parameters L/d, K and  $E_b/E_s$  (Poulos & Davis, 1980; Chin, 1988). However, slip at the pile-soil interface generally plays a dominant role in pile behaviour. Poulos & Davis (1980) present solutions that indicate the circumstances under which slip occurs along virtually the entire pile shaft. For typical normally consolidated or lightly-overconsolidated soils, if a surface pressure q is applied to the soil, full slip is likely to occur if

$$q > (0.3 - 0.5)\gamma'L$$
 (12)

where  $\gamma'$  is the submerged unit weight of soil and L is the embedded length of pile.

The soil surface movement required to develop full slip depends on the relative stiffness of the pile and the distribution of soil modulus and shaft resistance with depth. Values typically range between about 0.5% of diameter for relatively short stiff piles in soft soils, to 5% of diameter for relatively long piles.



Fig. 17. Influence of pile-soil slip on downdrag force and pile movement

In contrast to the case of conventional axial loading, the development of pile-soil slip is advantageous. It leads to reduced pile movement and downdrag forces as compared with the purely elastic interface condition. A typical example is shown in Fig. 17. Pile-soil slip starts when the soil surface movement is about 10 mm, and is almost complete at about 25 mm. Elastic theory would seriously overestimate the pile force and movement for soil movements in excess of about 20 mm.

Because of the dominant influence of pile-soil slip, the soil model used in the analysis of negative friction generally does not have a major effect on the solution. Fig. 18 shows a typical example in which three models, the elastic-plastic continuum, the elastic-plastic t-z model, and the hyperbolic t-z model, have been used, with the same values of initial tangent Young's modulus and shaft resistance. Before the development of full slip, the continuum model tends to give values of downdrag force and pile movement which are about 30% smaller than the other two methods, but as pile-soil slip develops, the differences become less. It will also be observed that the non-



Fig. 18. Influence of soil model on computed downdrag force in end-bearing pile

linear hyperbolic t-z model gives results which are quite close to those from the elastic-plastic t-z model (i.e. the details of the non-linearity appear to be relatively unimportant).

In contrast to conventional axial loading, group effects stemming from interaction among piles have a beneficial effect in the case of negative friction problems. As compared with a single isolated pile, there is a tendency for reduced downdrag forces and pile movements in the group, especially for inner piles within the group (Chin, 1988; Kuwabara & Poulos, 1989). For inner piles, the downdrag force is reduced to a small fraction of the value for an isolated single pile and even the corner piles have a maximum force only about one-third of the single pile value. Thus, in designing a group of piles to withstand the effects of negative friction, it would be extremely conservative to assume that all piles in the group are subjected to the downdrag force which would be developed in a single pile.

#### Piles in expansive soils

Because expansive soils are frequently relatively stiff, elastic interface conditions are more likely to be relevant for piles in expansive soils than for piles in soft clay subjected to downdrag. For purely elastic interface conditions, and a relatively short pile in a uniform soil, Fig. 19 shows the effect of the relative depth of soil movement  $z_s/L$ and the pile stiffness factor K, on the relative pile head movement  $\rho_p/S_0$  (where  $S_0$  is the soil surface movement). The main factor that influences pile head movement is the depth of soil movement; the influence of pile compressibility is relatively small.



Fig. 19. Elastic solutions for pile movement in expansive soil—uniform pile diameter



Fig. 20. Effect of pile diameter—pile in expansive soil: (a) pile top movement; (b) tensile stress in pile

For a given length of pile, the diameter has relatively little effect on the movement of the pile. Fig. 20 shows an example of the variation of pile head movement and maximum tensile stress in the pile as a function of diameter. Solutions for both a purely elastic interface, and with consideration of pile-soil slip, are shown. The elastic solutions are conservative and give larger movements and stresses than the solutions incorporating slip. For pile diameters between about 0.2 and 2.0 m, the pile head movement is almost constant when pile-soil slip is accounted for. The maximum tensile stress in the pile increases with decreasing diameter, but does not reach a significant level until the diameter is less than 0.2 m. The important implication of these results is that it is possible to use relatively small-diameter piles to suppress foundation movements in expansive soils. This point was also made by Donaldson (1967) who described the successful use of smalldiameter piles to support brick buildings on expansive soils.

The use of an enlarged base tends to reduce the pile movement, but this reduction is only effective for pile length to diameter ratios less than 15. The maximum effect appears to be obtained when the bell is located at or just below the active zone of soil movement.

In a group of piles in expansive soil, there is a tendency for the pile forces and movements to be reduced because of interaction among the piles. This theoretical tendency is supported by measurements reported by Blight (1984) on instrumented piles within a seven-pile group.

### Piles subjected to cyclic axial loading

Laboratory test data on model piles suggest that one of the most important effects of cyclic loading is to cause a reduction (or degradation) of pile shaft resistance. This degradation depends on the amplitude of cyclic displacement to which the pile is subjected, and on the number of cycles. There appears to be a threshold cyclic displacement below which no degradation occurs, this threshold being of the same order as the displacement to cause pile-soil slip under static loading. As the cyclic displacement increases beyond this value, there is an increasing loss of skin friction; this loss also increases as the number of cycles increases. There are currently insufficient experimental data to define clearly whether the degradation of shaft resistance depends on the absolute or the *relative* cyclic displacement, but some data from model and field tests on grouted piles in calcareous sediments (Lee, 1988) suggests the latter. Fig. 21 summarizes this data and plots the degradation factor for shaft resistance as a function of the normalized cyclic slip displacement (i.e. the total cyclic displacement minus the threshold value) divided by pile diameter d. A reasonably consistent relationship is obtained for model pile diameters of between 24 and 77 mm, and field pile diameters between 440 and 589 mm. Fig. 21 shows that major reductions in shaft resistance can occur for this case, with the possibility of the value after severe cyclic loading being less than 10% of the initial value for static loading.

Using data such as that in Fig. 21, it is possible to analyse the behaviour of a pile subjected to cyclic axial loading. Some findings from these analyses are presented here.

A useful means of portraying the behaviour of piles subjected to cyclic loading is by means of a



Fig. 21. Effect of normalized cyclic slip displacement on  $D_f$  with different pile diametes (after Lee, 1988)

cyclic stability diagram (Poulos, 1988b). This is a normalized plot of mean load against cyclic load (each being divided by the static compressive load capacity  $Q_c$ ), in which at least three regions can be identified

- (a) a cyclically stable region in which cyclic loading has no influence on the axial capacity of the pile
- (b) a cyclically metastable region in which cyclic loading causes some reduction of axial load capacity, but the pile does not fail within a specified number of cycles
- (c) a cyclically unstable region in which cyclic loading causes sufficient degradation for the pile to fail within a specified number of cycles of load.

In addition, Lee (1988) has identified a fourth zone, a serviceability loss zone, which lies within the metastable zone and in which excessive settlement of the pile develops within a specified number of cycles.

Figure 22 shows an example of a cyclic stability diagram for a 100 m long offshore grouted pile (Lee, 1988) in a soil whose Young's modulus and shaft resistance increase linearly with depth. In this case, stable behaviour will occur only if the cyclic load is less than about 20% of the ultimate compressive static load. Comparisons between theory and model and field test data, presented by Poulos (1988b) and Lee (1988), show quite good agreement and tend to confirm the validity of the concept of the cyclic stability diagram.



Fig. 22. Typical cyclic stability diagram for offshore pile; N = 100 cycles (lee, 1988)

It has been observed that two-way cyclic loading (about zero mean load) has a more severe effect on piles than one-way cyclic loading (for which the minimum load is zero-i.e. the cyclic load equals the mean load). This observation can readily be verified from the cyclic stability diagram. Referring to Fig. 22, for two-way cyclic loading, the unstable zone for N = 100 cycles begins at a normalized cyclic load of about 0.57; this is also the maximum load that can be sustained under two-way cyclic loading for 100 cycles. For one-way cycling  $(P_c = P_0)$ , the unstable zone is reached when  $P_c/Q_c = 0.46$ , so that the maximum load that can be sustained under one-way loading conditions is 0.92  $Q_c$ , substantially greater than the value of 0.57  $Q_c$  for one-way loading.

As piles become shorter or stiffer, the positions of the boundaries between the various zones alter; both the stable and unstable zone boundaries increase, but the metastable zone shrinks. Fig. 23 shows the effect of pile length on the zone boundaries for a 1.5 m dia. driven steel tube pile in clay (Poulos, 1988b). For long compressible piles, there is a large metastable zone. Such piles exhibit a gradual decrease in load capacity as the cyclic load level is increased. In contrast, short stiff piles have a very limited metastable zone, and may fail abruptly after small increases in cyclic load above the stable zone. It may thus be considered that long compressible piles exhibit a



Fig. 23. Effect of pile length on stability diagram zone boundaries. Driven pile in clay (Poulos, 1988b)

ductile cyclic behaviour, while short stiff piles show a brittle cyclic behaviour.

Group effects may influence the cyclic behaviour of the piles, but to a relatively minor extent. Hewitt (1988) has performed analyses of a single pile, and four- and eight-pile groups, using the Matlock & Foo model for degradation of shaft resistance. For two-way loading of relatively stiff piles, these results are shown in Fig. 24 together with laboratory data on model piles in kaolin. The theory shows that the maximum cyclic load



Fig. 24. Influence of number of piles on failure under two-way cyclic loading

that can be sustained decreases as the number of piles increases. The experimental results for 10 cycles are in good agreement with the theoretical curve.

# ESTIMATION OF GEOTECHNICAL PARAMETERS

The most significant parameters required for many of the category 2A, 2B, 3A and 3B analyses of pile behaviour under static loading are as follows

- (a) the shaft resistance  $f_s$
- (b) the end bearing resistance  $f_{\rm b}$
- (c) Young's modulus of the soil  $E_s$
- (d) Poisson's ratio of the soil  $v_s$

(e) the hyperbolic curve-fitting constant  $R_{\rm f}$ .

For prediction of cyclic axial response, a number of other parameters are required, and some limited data on these is presented by Poulos (1988c). Attention will be concentrated here on the above parameters, and in particular on  $f_s$ ,  $f_b$ and  $E_s$ .

For calculation of axial pile load capacity,  $f_s$  and  $f_b$  must be estimated as accurately as possible. For the calculation of settlement resulting from direct axial loading, the theoretical solutions reveal that the choice of an appropriate value of  $E_s$  is generally crucial, unless the piles are long and compressible. For piles in soil subjected to external movement, the pile behaviour is generally much less dependent on  $E_s$  and, provided that the soil movement is known,\* an approximate estimate of  $E_s$  may be adequate, although reasonable estimates of shaft and end-bearing resistance are desirable.

### Methods of determining parameters

For evaluating the parameters for static pile response, a number of methods can be contemplated, including

- (a) laboratory testing
- (b) appropriate interpretation of field pile load tests
- (c) empirical correlations with laboratorydetermined parameters
- (d) empirical correlations with the results of in-situ test data.

Conventional laboratory tests, such as triaxial or oedometer tests, are generally not suitable for direct measurement of the soil Young's modulus as they do not follow, even approximately, the stress path which the soil adjacent to the pile follows. Laboratory model pile tests may overcome this deficiency to some extent, but may not accurately reflect the behaviour of prototype piles because of the presence of scale effects, particularly for piles in sand. Some potential exists for more sophisticated tests such as the constant normal stiffness (CNS) direct shear test (Johnston & Lam, 1984; Ooi & Carter, 1987), and this type of test has been used in the design of grouted piles in offshore carbonate sediments (Johnston *et al.*, 1988). However, the direct utilization of laboratory tests for pile design is infrequent in practice, and still requires further research before it can be applied with confidence.

The most reliable means of determining  $f_s$  and  $E_s$  is by backfiguring from the results of pile load tests. Methods for interpreting the pile load test data have been detailed by Poulos & Davis (1980) and Stewart & Kulhawy (1981), among others. Such methods are particularly effective if the pile is instrumented so that details of the load transfer along the pile shaft are available; it is then possible to determine detailed distributions of soil modulus and limiting pile-soil friction along the pile shaft.

A variant of the pile loading test is the pile section test. This involves the testing of a series of relatively short rigid sections at different depths, in order to determine the distribution of limiting pile-soil friction and soil modulus with depth. In such tests, the sections must be installed in a similar manner to the prototype pile in order to obtain appropriate data. Examples of section tests have been reported by Hyden *et al.* (1988), and Williams & van der Zwaag (1988); in both cases, tests were carried out on grouted pile sections in marine calcareous sediments.

In most practical situations, it is not possible to carry out such testing, at least in the early stages of design. Resort is frequently made to correlations between the pile design parameters and laboratory or field test data.

# Shaft resistance $f_s$

Tables 5 and 6 summarize available methods for determining the shaft resistance  $f_s$  from laboratory strength data, for both driven and bored piles. Effective stress approaches can be used for all soil types, whereas a total stress approach is still adopted commonly for piles in clay. The parameters  $\alpha$  and  $\beta$  (or K and  $\delta$ ) are usually obtained from empirical correlations, despite the fact that the effective stress  $\beta$  approach is fundamentally sound and falls into category 2.

A summary of some suggested correlations between  $f_s$  and the standard penetration resistance N are given in Table 7. Considerable varia-

<sup>\*</sup> The soil movement is treated here as an independent variable, although it will be influenced to some extent by the soil modulus.

Soil type	Equation	Remarks	Reference
Clay	$f_{\rm s} = \alpha c_{\rm u}$	$ \begin{aligned} \alpha &= 1.0 \ (c_u \leqslant 25 \ \text{kN/m}^2) \\ \alpha &= 0.5 \ (c_u \geqslant 70 \ \text{kN/m}^2) \\ \text{Linear variation in between} \end{aligned} $	API (1984)
		$ \begin{array}{l} \alpha = 1 \cdot 0 \; (c_u \leqslant 35 \; \mathrm{kN/m^2}) \\ \alpha = 0 \cdot 5 \; (c_u \geqslant 80 \; \mathrm{kN/m^2}) \end{array} $	Semple & Rigden (1984)
		Linear variation in between. Length factor applies for $L/d > 50$	
		$\alpha = \left(\frac{c_{\mathbf{u}}}{\sigma_{\mathbf{v}}'}\right)^{0.5} \left(\frac{c_{\mathbf{u}}}{\sigma_{\mathbf{v}}'}\right)^{-0.5}  \text{for}  \left(\frac{c_{\mathbf{u}}}{\sigma_{\mathbf{v}o}'} \leqslant 1\right)$	Fleming et al. (1985)
		$\alpha = \left(\frac{c_{u}}{\sigma_{v}}\right)^{0.5} \left(\frac{c_{u}}{\sigma_{v}'}\right)^{-0.25}  \text{for}  \left(\frac{c_{u}}{\sigma_{vo}'} \ge 1\right)$	
	$f_{\rm s}=eta\sigma_{\rm v}'$	$\beta = (1 - \sin \phi') \tan \phi' (\text{OCR})^{0.5}$	Burland (1973) Meyerhof (1976)
Silica sand	$ \begin{array}{c} f_{\rm s} = \beta \sigma_{\rm v}' \\ (f_{\rm s} \ge f_{\rm slim}) \end{array} $	$\beta = 0.15 - 0.35$ (compression) 0.10 - 0.24 (tension)	McClelland (1974)
		$\beta = 0.44 \text{ for } \phi' = 28^{\circ} \\ 0.75 \text{ for } \phi' = 35^{\circ} \\ 1.2 \text{ for } \phi' = 37^{\circ} \end{cases}$	Meyerhof (1976)
		$\beta = (K/K_0) \cdot K_0 \cdot \tan(\phi \cdot \delta/\phi)$ $\delta/\phi \text{ depends on interface materials}$ (range 0.5-1.0); $K/K_0 \text{ depends on installation}$ method (range 0.5-2.0). $K_0 = \text{coefficient of earth pressure}$ at rest, and is a function of OCR	Stas & Kulhawy (1984)
Uncemented calcareous sand	$f_{\rm s} = \beta \sigma_{\rm v}'$	$\beta = 0.05 - 0.1$	Poulos (1988d)

Table 5. Shaft resistance f, for driven piles, determination from laboratory strength data

tions occur in these correlations, particularly for bored and cast-in-place piles.

Figures 25 and 26 show values of  $f_s$  correlated with static cone resistance  $q_c$ . These relationships have been developed by the Author from the correlations suggested by Bustamante & Gianeselli (1982) and cover a wide range of pile types in both clay and silica sand. The classification of these pile types is shown in Table 8. It should be emphasized that several other correlations have been proposed and that wide variations exist between some of these. Fig. 27 shows an example of this variability, for driven piles in silica sand. The potential inaccuracy of shaft capacity prediction using category 1 correlations, especially for loose sands, is clearly demonstrated.

Schmertmann (1975; 1978) proposes a different approach to the utilization of cone data, whereby the pile shaft resistance is related to the measured sleeve resistance of the penetrometer. Corrections are applied, depending on soil type, pile type, relative pile length, and depth below the surface. Robertson *et al.* (1985) have found the method proposed by Schmertmann to provide a more reliable prediction than the direct correlation to  $q_c$ , when applied to piles in a clayey silt.

Extensive correlations have been developed in France between shaft resistance and the limit pressure  $p_1$  deduced from pressuremeter measurements. Based on the results of over 300 pile load tests at more than 100 sites, Bustamante *et al.* (1987) have proposed correlations, similar in nature to those with the CPT in Figs 25 and 26.

Correlations such as those outlined must always be employed with caution, as a number of other factors may also influence shaft resistance e.g. the presence of overlying layers (Tomlinson, 1977).

Soil type	Equation	Remarks	Reference
Clay	$f_{\rm s} = \alpha c_{\rm u}$	$\alpha = 0.45$ (London clay)	Skempton (1959)
		$\alpha = 0.7$ times value for driven displacement pile	Fleming et al. (1985)
	$f_{\rm s} = K \tan \delta \sigma_{\rm v}'$	K is lesser of $K_0$ or $0.5(1 + K_0)$	Fleming et al. (1985)
		$K/K_0 = 2/3$ to 1; $K_0$ is function of OCR; $\delta$ depends on interface materials	Stas and Kulhawy (1984)
Silica sand	$f_{\rm s}=eta\sigma_{ m v}'$		Meyerhof (1976)
		$\beta = F \tan (\phi' - 5^{\circ})$ where $F = 0.7$ (compression) & 0.5 (tension)	Kraft & Lyons (1974)
Uncemented calcareous sand	$f_{\rm s} = \beta \sigma_{\rm v}' (f_{\rm s} \ge f_{\rm slim})$	$\beta = 0.5 \text{ to } 0.8$ $f_{\text{slim}} = 60 \text{ to } 100 \text{ kN/m}^2$	Poulos (1988d)

Table 6. Shaft resistance  $f_n$  for bored piles, determination from laboratory strength data

# End bearing resistance $f_{\rm b}$

Table 9 summarizes the two usual methods used for assessment of the end bearing resistance of piles using laboratory data. A total stress approach is almost invariably used for piles in clay, whereas an effective stress approach is used for piles in sand. Two main problems arise in the latter case

(a) some experimental evidence suggests that a limiting value of  $f_b$  may occur when the pile is



Fig. 25. Design values of shaft resistance for piles in clay (based on Bustamante & Gianeselli, 1982)

embedded more than 10 to 20 diameters; no entirely satisfactory method of theoretical analysis has been developed to take this into account and an empirical upper limit to  $f_b$  is usually specified

(b) the theoretical bearing capacity factor  $N_q$  is very sensitive to the angle of internal friction  $\phi'$ ; for values of  $\phi'$  in excess of about 35°,



Fig. 26. Design values of shaft resistance for piles in sand (based on Bustamante & Gianeselli, 1982)

Pile type	Soil type	α	β	Remarks	Reference
Driven displacement	Cohesionless	0	2.0	$f_s$ = average value over shaft $\overline{N}$ = average SPT along shaft Halve $f_s$ for small displacement pile	Meyerhof (1956) Shioi & Fukui (1982)
	Cohesionless & cohesive	10	3.3	Pile type not specified $50 \ge N \ge 3$ $f_s \ge 170 \text{ kN/m}^2$	Decourt (1982)
	Cohesive	0	10		Shioi & Fukui (1982)
Cast in place	Cohesionless	30	2.0	$f_{\rm s} \ge 200 \ \rm kN/m^2$	Yamashita et al.
		0	5.0		(1987) Shioi & Fukui (1982)
	Cohesive	0	5.0	$f_{\rm s} \ge 150 \ \rm kN/m^2$	Yamashita et al.
		0	10-0		(1982) Shioi & Fukui (1982)
Bored	Cohesionless	0	1.0		Findlay (1984) Shioi & Fukui (1982)
		0	3.3		Wright & Reese (1979)
	Cohesive	0	5.0		Shioi & Fukui (1982)
	Cohesive	10	3.3	Piles cast under pentonite $50 \ge N \ge 3$ $f_s \ge 170 \text{ kN/m}^2$	Decourt (1982)
	Chalk	-125	12.5	30 > N > 15 $f_{\rm s} \ge 250 \text{ kN/m}^2$	After Fletcher & Mizon (1984)

Table 7. Correlations between shaft resistance f, and SPT value, with  $f_a = \alpha + \beta N \text{ kN/m}^2$ 

small changes in  $\phi'$  can theoretically lead to large changes in  $N_q$ , although the effects of soil compressibility are then more important and may reduce the dependence of  $N_q$  on  $\phi'$ .

Table 10 shows some empirical correlations between  $f_b$  and the standard penetration resistance in the vicinity of the pile tip. These correlations indicate that bored or cast-in-place piles develop a significantly smaller end-bearing resistance than do driven piles.

Bustamante & Gianeselli (1982) suggested correlations between  $f_b$  and the average cone penetration resistance value near the pile tip. The correlation factor to the latter value is between 0.3 and 0.55, depending on soil and pile type. These correlations contrast with procedures such as those proposed by Belcotec (1985) and De Ruiter & Beringen (1979), in which a factor of unity is applied to the average value (computed differently than in the Bustamante & Gianeselli approach). However, the latter approaches are confined to driven piles, whereas the Bustamante & Gianeselli approach is more general, simpler to apply, and probably more conservative.

Baguelin et al. (1986) and Bustamante et al. (1987) have related  $f_b$  to the pressuremeter limit pressure  $p_1$  by way of a factor  $k_p$  which depends on pile and soil types. For non-displacement piles,  $k_p$  is 1.2 for clays and silts and 1.1 for sands, whereas for driven piles,  $k_p$  is 1.8 for clays and silts, and ranges between 3.2 and 4.2 for sands.

Pile category	Type of pile
ΙΑ	Plain bored piles, mud bored piles, hollow auger bored piles, cast screwed piles Type I micropiles, piers, barrettes
IB	Cased bored piles Driven cast piles
IIA	Driven precast piles Prestressed tubular piles Jacked concrete piles
IIB	Driven steel piles Jacked steel piles
IIIA	Driven grouted piles Driven rammed piles
IIIB	High pressure grouted piles ( $d > 0.25$ m) Type II micropiles

 Table 8.
 Classification of pile types (Bustamante & Gianeselli, 1982)

Soil Young's modulus  $E_s$ 

Ideally, for piles in clay, a distinction should be made between the undrained Young's modulus, used for calculations of immediate or undrained settlement, and the drained Young's modulus, used for calculations of total settlement of a pile.

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However, for many clays, the difference between the drained and undrained modulus values is not great and the approximate nature of most correlations makes such a distinction impractical. It is therefore suggested that the correlations presented in this Paper should be considered to apply to

Table 9. End bearing capacity of pile tip,  $f_b$ , determination from laboratory data

Soil type	Equation	Remarks	Reference
Clay	$f_{\rm b} = N_{\rm c} c_{\rm ub}$	$N_{c} = 9 \text{ for } L/d \ge 3$ $c_{ub} = \text{value of } c_{u} \text{ in }$ vicinity of pile tip	Skempton (1959)
Silica sand*	$f_{\rm b} = N_{\rm q} \sigma_{\rm v}'$	$N_q = 40$	API (1984)
	≁ Jblim !	$N_{ ext{q}}$ plotted against $\phi'$	Berezantzev et al. (1961)
		$N_q$ related to $\phi'$ , relative density and mean effective stress	Fleming <i>et al.</i> (1985)
		$N_q$ from cavity expansion theory, as a function of $\phi'$ and volume compressibility	Vesic (1972)
Uncemented	$f_{\rm b} = N_{\rm q}  \sigma_{\rm v}'$	$N_q = 20$	Datta et al. (1980)
calcareous sand	₽ J <sub>blim</sub>	Typical range of $N_q = 8-20$	Poulos (1988d)
		$N_{q}$ determined for reduced value of $\phi'$ (e.g. 18°)	Dutt and Ingram (1984)

\* For silica and calcareous sands, the above expressions apply for driven piles only.

† Typical limiting values  $f_{\text{blim}}$  range from 10–15 MN/m<sup>2</sup> for silica sand, and 3–5 MN/m<sup>2</sup> for calcareous sand; the latter value depends on soil compressibility (Nauroy *et al.*, 1986).



Fig. 27. Example of variations between correlations for shaft resistance against CPT-driven piles in sand: (a)  $q_c = 6 \text{ MN/m}^2$ ; (b)  $q_c = 10 \text{ MN/m}^2$ ; (c)  $q_c = 20 \text{ MN/m}^2$ 

the drained Young's modulus. For all soil types, a further distinction needs to be made, between the tangent value of Young's modulus (if a non-linear interface model is being used) and the secant value of Young's modulus (if a purely linear analysis is being used). Again, it is often difficult to make such distinctions with rough empirical correlations and, unless specified, the Young's modulus referred to here will be a secant value, relevant for normal working load levels of between one-third to one-half of the ultimate load capacity.

For piles in clay, Young's modulus has been correlated often with laboratory-measured undrained shear strength,  $c_u$ . Some of these correlations are shown in Fig. 28, and a feature of this figure is the wide spread of correlations. Possible reasons for this spread might include differences in the method of determining  $c_u$ , differences in the method of determining the modulus values, differences in the load level at which the modulus was determined, differences in the overconsolidation ratio of the clay between different tests, and differences between the clay types. Callanan & Kulhawy (1985) find that values of  $E_s/c_u$ 



Fig. 28. Correlations for soil modulus for piles in clay (after Callanan & Kulhawy, 1985)

generally range between 200 and 900, with an average value of about 500. These values apply to piles with a length-to-diameter ratio in excess of about 15. For shorter piles, the upper range of  $E_s/c_u$  may be greater because of the possible effects of fissuring, desiccation and over-consolidation of the clay near the surface.

Some correlations between Young's modulus and standard penetration test number are summarized in Fig. 29, and show alarming variability. At least some of this variability may be attributed to differences in the determination or definition of the SPT value, but it is clear that the potential for selection of inappropriate values of  $E_s$  is great.



Fig. 29. Comparison between correlations for soil modulus driven piles in sand

Pile type	Soil type	K	Remarks	Reference
Driven displacement	Sand	0.45	N = average SPT value in local failure zone	Martin <i>et al.</i> (1987)
	Sand	0.40		Decourt (1982)
ļ	Silt, sandy silt	0.32		Martin <i>et al.</i> (1987)
ļ	Glacial coarse to fine silt deposits	0.22		Thorburn & MacVicar (1971)
	Residual sandy silts	0.25		Decourt (1982)
!	Residual clayey silts	0.20		Decourt (1982)
	Clay	0.20		Martin <i>et al.</i>
	Clay	0.12		Decourt (1982)
	All soils	0.30	For $L/d \ge 5$ If $L/d < 5$ , K = 0.1 + 0.04 L/d (closed-end piles) or K = 0.06 L/d (open-ended piles)	Shioi & Fukui (1982)
Cast in	Cohesionless		$f_{\rm b} = 3.0 \rm MN/m^2$	Shioi & Fukui
place		0.15	$f_{\rm b} \Rightarrow 7.5 \ {\rm MN/m^2}$	Yamashita <i>et al.</i> (1987)
	Cohesive		$f_{b} = 0.09 (1 + 0.16z)$ where z = tip depth (m)	Yamashita et al. (1987)
Bored	Sand	0.1		Shioi & Fukui (1982)
	Clay	0.15		Shioi & Fukui (1982)
	Chalk	0·25 0·20	N < 30 N > 40	Hobbs (1977)

Table 10. Correlations between end bearing resistance  $f_b$  and SPT value;  $f_b = KN MN/m^2$ 

Table 11 shows some suggested correlations between  $E_s$  and cone penetration resistance  $q_c$ , and, as with most of the other correlations, the range is large. Two correlations for initial tangent modulus  $E_{st}$  are shown, both being derived from dynamic triaxial tests but believed to be relevant for piles.

Correlations between  $E_s$  and pressuremeter data have not been extensively developed, although Frank (1985) suggests that the initial tangent modulus can be taken as the initial tangent modulus of the expansion curve from the self-boring pressuremeter. Most of the correlations with pressuremeter data have been made with respect to the axial load transfer curves for a pile. A summary of a number of these correlations, for both pile shaft and pile tip responses, is given by Frank (1985).

### Poisson's ratio v<sub>s</sub>

Poisson's ratio of the soil is a necessary input parameter into analyses that involve elastic continuum theory, but its effect is generally quite minor when the solutions are expressed in terms of Young's modulus of the soil. For saturated

Soil type	Correlation	Remarks	Reference
Clay and silts	$E_{\rm s}^{*} = 21.0q_{\rm c}^{1.09}$	Various pile types $E_{\rm s}$ and $q_{\rm c}$ in MN/m <sup>2</sup>	Christoulas (1988)
	$E_{\rm s}=15q_{\rm c}$		Poulos (1988c)
Silica sands	$E_{\rm s}=lpha q_{\rm c}$	α = 20-40	Milovic & Stevanovic (1982)
		$\alpha = 5$ (normally- consolidated sands) $\alpha = 7.5$ (over- consolidated sands)	Poulos (1988c)
	$E_{\rm st}^{\dagger} = 53q_{\rm c}^{0.61}$	$E_{\rm st}$ and $q_{\rm c}$ in MN/m <sup>2</sup> Dynamic modulus value	Imai & Tonouchi (1982)
Unspecified	$E_{\rm st} = \alpha q_{\rm c}$	$\alpha = 24-30$ Dynamic modulus value	Holeyman (1985)
	$E_{\rm s}=10.8+6.6q_{\rm c}$	$E_{\rm s}$ and $q_{\rm c}$ in MN/m <sup>2</sup> (for $q_{\rm c} > 0.4$ MN/m <sup>2</sup> )	Verbrugge (1982)

Table 11. Correlations between soil Young's modulus E, and CPT value—driven piles

\*  $E_s$  = secant Young's modulus.

 $+ E_{st} = initial tangent Young's modulus.$ 

clays under undrained conditions,  $v_s$  can be taken as 0.5. For clays under drained conditions,  $v_s$  generally lies within the range  $0.35 \pm 0.05$ , whereas for silica sands,  $v_s$  is usually within the range  $0.3 \pm 0.1$ . Lower values, within the range  $0.15 \pm 0.1$ , are applicable for many marine calcareous sediments.

### Hyperbolic curve fitting constant $R_{\rm f}$

If a hyperbolic interface model is used,  $R_f$  defines the degree of non-linearity and can range between 0 (an elastic-perfectly plastic response) and 1.0 (an asymptotic hyperbolic response in which the limiting pile-soil stress is never reached). Limited experience suggests that different values of  $R_f$  should be used for shaft and pile tip elements. For the shaft, there is a relatively small amount of non-linearity, and values of  $R_f$  in the range 0-0.5 may be appropriate. In contrast, the pile tip response is often highly non-linear, and it is suggested that a value of  $R_f$  of about 0.9 may give a reasonable fit with observed behaviour.

In much of the published theoretical work to date, elastic-plastic response has been assumed (i.e.  $R_f = 0$  for both shaft and tip). As pointed out by Poulos & Davis (1980) and Frank (1985), displacements at load levels approaching failure are therefore often seriously underestimated. Fortunately however, at normal working loads, the effects of non-linearity are not great and adequate

predictions of settlement are often achieved with elastic or elastic-plastic theory (Fig. 11).

# DESIGN CHARTS FOR PILES AND PILE GROUPS

Dimensionless category 2 solutions for pile settlement, such as those presented by Butterfield & Banerjee (1971), Randolph & Wroth (1978), Banerjee (1978), Poulos & Davis (1980) and Butterfield & Douglas (1981) are useful for preliminary design purposes and can often form the basis for final design calculations. In applying such solutions, it is necessary for the user to determine the most appropriate simplified soil profile and the values of Young's modulus for the soil in this profile. These determinations may be difficult in the early stages of a project when little quantitative data is available, and it is therefore of value to develop design charts based on hypothetical but realistic soil data. A limited series of such charts is presented in this section for driven and bored piles in both silica sand and clay profiles. The geometrical and soil parameters are defined in Fig. 30. The assumed parameters for the sand and clay profiles are summarized in Table 12 and are based on the Author's judgement and the correlations presented in the preceding section.

For all the charts presented, Young's modulus of the pile,  $E_p$ , has been taken as 30000 MN/m<sup>2</sup>. This value is representative of concrete piles and



Fig. 30. Definition of soil modulus distributions assumed for design charts

also of the effective modulus of most steel tube piles, for which the section area lies between 10 and 20% of the gross cross-sectional area.

# Settlement of single piles

The charts for single pile settlement have been developed from the closed-form solution developed by Randolph & Wroth (1978), which can be evaluated conveniently by means of a computer spreadsheet program.

Figures 31-34 present theoretical relationships between the pile head settlement per unit load S/P and the pile length L, for driven and bored piles having a diameter in the range 0.4-0.7 m.

Table 12. Parameters used to derive design charts\*



Fig. 31. Design chart for settlement of driven piles in clay (diameter =  $0.5 \pm 0.1$  m)

Similar charts may be developed for other diameters. In each case, a range of soil stiffness or density, representing the likely limits of practical conditions, is considered. Also shown on these charts are observed flexibilities (at average working load levels of  $40 \pm 10\%$  of ultimate load) from published field load tests. Several have been taken from the useful compilation prepared by Kulhawy *et al.* (1982). The observed values of S/P

Pile type	Soil type	$E_{so}$ : MN/m <sup>2</sup>	m: MN/m <sup>3</sup>	$\frac{E_{\rm b}}{E_{\rm SL}}$	$\mu = \frac{E_{\rm sm}}{E_{\rm s}}$	v <sub>s</sub>	$f_s$ : kN/m <sup>2</sup>	$f_b$ : MN/m <sup>2</sup>
Driven	soft clay medium clay stiff clay hard clay loose sand medium-dense sand very dense sand	0 20 42  0 0 0	$ \begin{array}{c} 0.7\\0\\-\\1.5\\4.0\\8.0\end{array} $	$ \begin{array}{c} 1 \cdot 0 \\ 1 \cdot 0 \\ - \\ 4 \cdot 0 \\ 2 \cdot 0 \\ 1 \cdot 5 \end{array} $	3.0 4.0 5.0  5.0 3.0 	$ \begin{array}{c} 0.5 \\ 0.5 \\ 0.5 \\ \\ 0.3 \\ 0.3 \\ \\ \end{array} $	25 45 70 —	0·2 0·5 
Bored	medium clay stiff clay hard clay loose sand medium-dense sand very dense sand	30 80 150 0 0 0	0 0 1·0 3·0 6·0	$ \begin{array}{c} 0.5 \\ 0.5 \\ 1.0 \\ 0.5 \\ 1.0 \\ 1.0 \\ 1.0 \end{array} $	4·0 2·0 — 3·0 1·5	0.5 0.5 	  	

Table 12. Farameters used to derive design charts

\* Fig. 30 gives definition of parameters. In all cases,  $s_i/d = 3.0$ .



Fig. 32. Design chart for settlement of driven piles in sand (diameter =  $0.5 \pm 0.1$  m)

are generally consistent with the theoretical values, although in some cases, they are less than the computed value for the stiffest or densest soil condition. Thus, it may be expected that the design charts will occasionally give a conservative estimate of settlement.



Fig. 33. Design chart for settlement of bored piles in clay (diameter =  $0.6 \pm 0.1$  m)



Fig. 34. Design chart for settlement of bored piles in sand (diameter =  $0.5 \pm 0.1$  m)

It is relevant to note that Frank (1985) has summarized practical experience with isolated piles and has suggested that, under the design load, the following settlements are typical

- (a) for driven piles: 0.9% of diameter (range 0.8– 1.2%)
- (b) for bored piles: 0.6% of diameter (range 0.3-1%).

A recent statistical analysis reported by Briaud & Tucker (1988) suggested that there is a 95% probability that the pile head settlement at half the ultimate load will be 1.25% of the pile diameter or less.

# Settlement of pile groups

Following the approach adopted for single piles, charts have been developed to enable a rapid estimate to be made of group settlements for both driven and bored piles in various soil types. The most convenient form of presentation is in terms of the group settlement ratio  $R_s$  and, more specifically, the exponent  $\omega$  of the settlement ratio in equation (11). An elastic boundary element analysis, by means of the computer program DEFPIG, has been used to obtain the solutions, with allowance being made for the higher stiffness of the soil between the piles than at the pile-soil interface (Poulos, 1988a).





Fig. 36. Settlement ratio exponent  $\omega$ , bored piles, s/d = 3: (a) medium clay; (b) stiff clay; (c) medium sand; (d) very dense sand

The design charts are shown in Figs 35 and 36 as plots of the settlement ratio exponent  $\omega$ against pile length for four soil types and for driven and bored friction piles of lengths between 10 and 30 m. The centre-to-centre spacing between the piles is assumed to be 3 diameters. The average value of  $\omega$  for square groups of between 4 and 25 piles, and for diameters of 0.3 m and 0.6 m, is plotted, together with the range of theoretical values of  $\omega$  for each pile length. For piles in clay,  $\omega$  is almost independent of pile length, whereas for the piles in silica sand,  $\omega$ decreases as the pile length increases. The range of values of  $\omega$  is narrow for shorter pile lengths, but increases as the pile length increases. Over the entire range of pile and soil types,  $\omega$  varies between 0.41 and 0.51 for 10 m long piles, and between 0.22 and 0.47 for 30 m long piles. There is therefore a clear implication that the use of these design charts is likely to be less accurate for long piles than for short piles.

To illustrate the use of the design charts for both single piles and pile groups, consider the case of a group of eight driven piles (in a  $4 \times 2$ configuration) each 20 m long and 0.6 m dia., in a deep layer of soft clay. The centre-to-centre spacing between the piles is 1.8 m. An estimate is required of the settlement of the group under a total load of 3.2 MN, which is equivalent to a safety factor of about 2.5 on the ultimate capacity.

From Fig. 31, for 20 m piles in soft clay, the value of S/P is about 14.0 mm/MN. Thus, at the average working load of  $3 \cdot 2/8 = 0.4$  MN, the settlement of a single pile would be  $14 \cdot 0 \times 0.4 = 5 \cdot 6$  mm. From Fig. 35, the average settlement ratio exponent  $\omega$  for 20 m long driven piles in soft clay is about 0.45. Thus, for an 8-pile group,  $R_s = 8^{0.45} = 2 \cdot 55$ . Therefore, the group settlement is estimated to be  $2 \cdot 55 \times 5 \cdot 6 = 14 \cdot 3$  mm.

# Ultimate load capacity of single piles

Although this section focuses on pile settlement, it is relevant to remark that design charts may also be developed for pile load capacity. Such charts are shown for driven piles in Figs 37–39 and are classified as category 1 charts because they are based on the empirical values of shaft and end bearing resistance shown in Table 12. Fig. 37 plots the ultimate shaft resistance against pile length for four pile diameters and three classifications of clay condition, soft, firm and hard. Figs 38 and 39 plot the pile tip cap-



Fig. 37. Design charts for ultimate shaft capacity of driven piles in clay: (a) d = 0.3 m; (b) d = 0.6 m; (c) d = 0.9 m; (d) d = 1.2 m



Fig. 38. Design chart for ultimate tip capacity of driven piles in clay

acity, as a function of pile tip diameter, for tips bearing in either clay or sand. Charts such as these are perhaps over-simple and must be used with due caution. Nevertheless, they can give a useful preliminary appreciation of the pile length and diameter requirements to develop a specified ultimate load capacity.

# CASE STUDIES

In predicting the behaviour of pile foundations, the geotechnical engineer is faced with a number of decisions, including

- (a) the method of analysis, and the associated soil model, to be used
- (b) the way in which the soil profile can be simplified and idealized for the analysis
- (c) the geotechnical parameters to be used.

The influence of these decisions on the predicted pile performance will be examined with respect to two published case histories, one involving an instrumented single pile, and the second involving a group of instrumented piles. In each case, theoretical calculations will be presented to illustrate the sensitivity of the predicted behaviour to the above factors, and these calculations will be compared with the observed behaviour. As an example of the application of theory to practice without the prior benefit of performance measurements, the results of a recent 'class A' prediction (Lambe, 1973) made by the Author will be presented and compared with observed behaviour.



Fig. 39. Design chart for ultimate tip capacity of driven piles in sand

# Case study 1—sensitivity of single pile performance calculations

The case analysed is a pile test described by Gurtowski & Wu (1984). The test pile was located at the site of the West Seattle Freeway in the USA. For the site under consideration (site A) the geotechnical profile is shown in Fig. 40 together with SPT data, which was the only quantitative geotechnical data available. The water table was located about 3 m below the surface.

The test pile was a 0.61 m wide octagonal prestressed concrete hollow pile with a plug at the tip, and was driven to a depth of about 30 m.



Fig. 40. Geotechnical data for site A (Gurtowski & Wu, 1984)

Aspect considered	Standard	Variations
Analysis method and soil model	Boundary element analysis, with elastic-plastic continuum soil model (with secant modulus)	Elastic continuum with secant modulus Elastic-plastic continuum with tangent modulus Hyperbolic continuum Elastic-plastic load transfer model, with secant modulus
Soil profile	Two-layer $0-125 \text{ m}  \bar{N} = 15$ $12.5 \text{ m}^+  \bar{N} = 40$	Homogeneous $(\overline{N} = 30)$ Gibson soil (N  varies from 0 to 60 at pile tip) Detailed profile, as indicated by SPT values
Soil parameters	$E_{s} = 4N \text{ MN/m}^{2}$ $f_{s} = 2N \text{ kN/m}^{2}$ $f_{b} = 0.4 \text{ MN/m}^{2}$	$E_{s} = 2.5N \text{ MN/m}^{2}$ $E_{s} = 7N$ $E_{s} = 36.8 + 1.04N$ $E_{s} = 7.5N - 94.5$
Pile modulus	$E_{\rm p}=35000~{\rm MN/m^2}$	$E_{\rm p} = 15000 {\rm MN/m^2} \\ E_{\rm p} = 50000 $

 
 Table 13. Analysis conditions and parameters for performance sensitivity evaluation

Note: N = SPT value.

A 'standard' set of conditions and parameters was chosen, and the effects of deviations from these standards were examined. The standard values chosen, and the deviations considered, are summarized in Table 13. The soil parameters are derived from the correlations presented in Tables 7, 10 and 12, and Fig. 29.

Figure 41 compares the measured loadsettlement behaviour and load distributions with those predicted from the standard analysis. The standard analysis predicts the pile head loadsettlement behaviour quite well, and also gives a good prediction of the pile load distribution at three different applied load levels. However, the head load against tip settlement relationship is not well predicted, indicating that perhaps too large a stiffness has been assigned to the soil at and beneath the pile tip.

In order to enable ready appreciation of the effects of deviations from the standard conditions, attention has been concentrated on the pile head settlement, the tip settlement, and the load at mid-depth of the pile, for an applied load of 1.8 MN, which would be a normal working load for this pile.

The influence of the soil model is shown in Fig. 42. In some cases,  $E_s$  is taken as the secant modulus, whereas in others, it is taken as the

initial tangent modulus. The pile head settlement and pile load at mid-depth are relatively insensitive to the soil model, whereas the tip settlement is influenced more. The two models involving the use of the initial tangent modulus give smaller tip movements than those using the secant modulus; all, however, predict a tip settlement which is much smaller than the measured value.

The effect of the soil profile idealization is shown in Fig. 43. The use of a homogeneous profile leads to a significantly smaller head settlement and mid-depth load than the other three profiles. The use of a detailed profile in which  $E_s$ and  $f_s$  vary as N varies gives a predicted behaviour which is little different from that for the twolayer profile. The indiscriminate assumption of a homogeneous soil profile can lead to inaccuracy in the predicted pile behaviour, whereas more realistic (but nevertheless simplified) modelling of the soil profile variation gives quite adequate results in this case.

Figure 44 shows the effect of the soil modulus correlation used for the analysis. The larger the soil modulus, the smaller are the settlements and the smaller is the load in the pile at mid-depth. The tip settlement is particularly affected by  $E_s$ . However, it is interesting to note that an increase in  $E_s$  from 2.5 N to 7 N MN/m<sup>2</sup> reduces the head



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(b) Pile Load Distribution

Fig. 41. Comparisons between observed and theoretical behaviour, pile test A (Gurtowski & Wu, 1984): (a) pile movements; (b) pile load distribution

settlement by less than 30%. This occurs because the pile is relatively compressible. In this case, because the piles are concrete, some uncertainty will be present in the choice of the pile modulus  $E_p$ . The importance of  $E_p$  is demonstrated in Fig. 45. Of all the factors considered, it has the greatest influence on the pile head settlement, although it has little influence on the tip settlement. The load distribution is also affected by the pile modulus, especially in the lower stiff soil layer.

The foregoing study therefore reveals that the head settlement of a relatively long compressible pile may be influenced as much by the pile modulus as by the soil model, the soil profile idealization, or the soil modulus correlation adopted. The soil modulus may have a substantial influence on the pile tip settlement, while the soil profile idealization can influence both load distribution and pile head settlement significantly. The effect of the soil model is not great, except for the tip settlement. Thus, in predicting the settlement of long compressible piles, it would seem desirable to attempt to determine the pile modulus as accurately as possible.

# Case study 2—sensitivity of pile group performance calculations

The sensitivity of theoretical predictions of pile group settlement and load distribution will be discussed with respect to the well-documented case study described by O'Neill *et al.* (1981; 1982).

Figure 46 summarizes the geotechnical data at the test site which was located at the University of Houston. The site consists of various layers of stiff to very stiff clay, and geotechnical data is available from standard penetration tests, cone penetration tests, pressuremeter tests, unconsolidated undrained triaxial tests, laboratory consolidation tests, and seismic cross-hole tests.

Vertical load tests were performed on full-scale pile groups and single piles, with measurements being made of settlement and load distribution

	Load at Mid-Depth (M	l) Soil Model	lement (mm)	
0	0.5 1.0	1.5 (	) 2	4 6
	• 1	Elastic Continuum (secant modulus)	•	1 <del>1- 1-</del>
	•	Elastic -Plastic Continuum (secant modulus)	•	1
	•	Elastic -Plastic Continuum (tangent modulus)	٠	Measured
	Measured	Hyperbolic Continuum (tangent modulus)	٠	I
	•	Load Transfer (secant modulus)	•	1



Fig. 42. Influence of soil model on predicted behaviour of single pile

for groups of nine, five and four piles, as well as two single piles. The piles were about 13 m long, 0.273 m dia. steel tubes with a 9.3 mm wall thickness.

In examining the influence of the method of analysis, only a limited number of methods were considered. The programs DEFPIG, PIGLET and GAPFIX were employed, with two DEFPIG analyses being carried out, the first a conventional analysis in which the soil modulus between the piles was assumed to be the same as that adjacent to the piles, and the second a modified analysis in which the soil between the piles was assumed to be stiffer than near the piles. The seismic cross-hole data were used to estimate the small-strain Young's modulus of the soil between the piles. For all analyses, the soil Young's modulus  $E_s$  (near the piles) was assumed to vary linearly with depth, according to the relationship  $E_s = 40 + 5.38z \text{ MN/m}^2$ , this relationship being based on

	Load at Mid-Depth (	(MN)	Soil Profile		Head Settle	ement	(mm)
0	0.5 1.0	1.	.5	0	2	4	6
	•		Homogeneous		' •	1	1
Γ	Measured		Linearly Varying		•	M	easured
Γ	•		Two-Layer		٠	1	
	•		Detailed		٠	1	



Fig. 43. Influence of soil profile idealization on predicted behaviour of single pile

	Load at Mid-Depth (MN)	Soil Modulus	Head Settlement (mm)			
0	0.5 1.0	1.5 Correlation 0	2	4	8	
Γ	•	$E_s=4N MN/m^2$	· •			
	Measured	E <sub>s</sub> =2.5N MN∕m <sup>2</sup>		ŕ		
	•	E <sub>s</sub> =7N MN∕m <sup>2</sup>	•	H	easured	
L	• 1	E <sub>s</sub> =36.8+1.04N MN/m <sup>2</sup>		•		
	•	E <sub>s</sub> =7.5N-94.5 MN/m <sup>2</sup>		•		

		Tip Settlement (mm)		
		0	1	2
Pile A Gurtowski and Wu (1984)	E <sub>s</sub> =4N MN∕m <sup>2</sup>	•	<b></b>	
	E <sub>s</sub> =2.5N MN/m <sup>2</sup>		٠	
Applied Load=1.8MN	E <sub>s</sub> =7N MN∕m <sup>2</sup>	•		- Measured
Elastic-Plastic Continuum	E <sub>s</sub> =36.8+1.04№ MN/m <sup>2</sup>		•	l
Two-Layer Soil Profile	E <sub>s</sub> =7.5N-94.5 MN/m <sup>2</sup>	•		

Fig. 44. Influence of soil modulus correlation on predicted behaviour of single pile



Fig. 45. Influence of pile modulus on behaviour of single pile: (a) head settlement; (b) tip settlement; (c) load distribution





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Fig. 47. Effect of analysis method on single pile and group settlements

the correlation  $E_s = 750 c_u$ . The values of  $c_u$  were obtained from the pressuremeter test data, the profile being simplified to vary linearly with depth from 53 kN/m<sup>2</sup> at the surface to 147 kN/m<sup>2</sup> at a depth of 13 m.

The calculated settlements of both the single pile and the nine-pile group are shown in Fig. 47. Also shown are the estimates of settlement derived from the category 2 design charts presented in this Paper for driven piles in stiff clay. There are some differences between the settlements derived from the different analyses, but these are not great. The single pile settlements given by the DEFPIG and GAPFIX analyses are close to the measured value, but the group settlements are overestimated by all approaches. The modified DEFPIG analysis gives the most satisfactory prediction of group settlement. The category 2 design charts overestimate both the single pile and group settlements, although this is to be expected as these charts are meant for design rather than prediction and therefore tend to be conservative.

For all the group configurations tested by O'Neill *et al.*, Fig. 48 compares the measured settlement ratios with those determined from the conventional and the modified DEFPIG analyses (Poulos, 1988a). The conventional analysis considerably overestimates  $R_s$ , whereas the modified analysis gives significantly closer agreement with the measured values.

The predicted distributions of load among the piles from the four analyses agree reasonably closely, although all give a less uniform distribution than was actually measured. Fig. 49 shows a detailed comparison between the pile load distributions with depth computed by the program GAPFIX and the measured values. While there are some differences near the pile head, the overall pattern of load transfer along each of the piles appears to be reasonably well predicted by the analysis.

The foregoing comparisons suggest that, given a set of common parameters, most of the methods considered give a similar prediction of group behaviour. However, for group settlement, better agreement is obtained with the modified DEFPIG analysis in which the greater stiffness of the soil between the piles is allowed for.

To examine the influence of the idealization of the soil profile on the group settlement, four different distributions of soil modulus with depth have been used, each being based on the undrained shear strength distribution obtained from the pressuremeter tests

- (a) the linearly varying distribution of  $E_s$  considered above
- (b) a homogeneous profile in which  $E_s = 75$  MN/m<sup>2</sup>



Fig. 48. Theoretical and measured group settlement behaviour (Tests of O'Neill et al., 1982)



Fig. 49. Comparison between computed and measured load distribution in 9 pile group test

- (c) a two layer profile in which the first layer extends to a depth of 8 m and has  $E_s = 55$ MN/m<sup>2</sup>, and the second extends to great depth and has  $E_s = 100$  MN/m<sup>2</sup>
- (d) a profile which follows the detailed distribution of  $c_u$  with depth, and in which  $E_s = 750$  $c_u$ .

The program DEFPIG was used for all calculations; no account was taken of increased soil stiffness between the piles.

Figure 50 shows the predicted settlements of the single pile and the nine-pile group. All four profiles give similar settlements, which agree well with the measured settlement for the single pile, but are about 50% too large for the group. There is also little variation among the solutions for the pile head load distribution, but in all cases the predicted load distribution is more non-uniform than that measured. It therefore appears that, in this particular case, the idealization of the soil profile is not a crucial factor in the prediction of pile group behaviour.

The influence of the method of determination

of the soil Young's modulus has been investigated by using six different procedures and employing the conventional DEFPIG analysis. Fig. 51 shows the predicted and measured settlements of the single pile and the nine-pile group. It is immediately apparent that the predicted settlements are more sensitive to  $E_s$  than to either the method of analysis or the soil profile idealization. The closest predictions are given by the correlations  $E_s = 750 c_u$  and  $E_s = 4N \text{ MN/m}^2$ , whereas  $E_s =$  $200 c_u$  and  $E_s = 20 q_c$  lead to group settlements. The correlations leading to large settlements also lead to more non-uniform distributions of pile head load.

For this case study, therefore, in which the piles were significantly shorter and less compressible than in the preceding case, the two factors which appear to be crucial in accurately predicting the group behaviour are

- (a) the magnitude of the soil Young's modulus  $E_s$
- (b) the greater stiffness of the soil between the piles than locally near the piles.



Fig. 50. Effect of soil profile idealization on single pile and group settlements



Calculated from conventional DEFPIG Program

Fig. 51. Effect of modulus correlation on single pile and group settlements

The general tendency with all conventional methods of group analysis is to overestimate the interaction and to predict a settlement which is too large and a load distribution which is too non-uniform. Such tendencies may not always be as significant as they appear to be for the case considered here.

# Case study 3—a class A prediction of pile behaviour

In conjunction with the Fifth Australia-New Zealand Conference on Geomechanics, held in Sydney in 1988, geotechnical engineers were invited to predict the load-settlement performance of two driven precast concrete piles at a site in Hemmant, Queensland, in Australia. After driving, each pile was left for several weeks and then tested to failure by static load testing procedures in accordance with the Australian Standard Piling Code. Sixteen engineers, including the Author, submitted 'class A' predictions based on static analyses, while four engineers undertook dynamic analyses using data from restriking tests performed four weeks after the static failure. A detailed description of the prediction exercise is given by Douglas (1989).

The piles were Balken piles, of high strength precast concrete, 275 mm square, cast in lengths of up to  $12 \cdot 2$  m, with mechanical jointing during driving. The piles were driven using a Banut 600 piling rig with a 5 t hydraulic hammer. Each pile was instrumented so that dynamic measurements could be taken during driving. Pile 1 was driven to a depth of 30.5 m and pile 2 to 25 m. Fig. 52

shows the soil profile and summarizes the available geotechnical data at the Hemmant site. The upper 26 m was essentially clay, underlain by very dense sand. Data were obtained from in situ static cone and dilatometer tests, and from laboratory triaxial tests. Values of undrained shear strength in the clays, derived from these three sources, agreed well.

In making his predictions, the Author derived the required parameters as follows.

- (a) The shaft resistance  $f_s$  in the clay soils was correlated with the undrained shear strength  $c_u$ , by way of the adhesion factor  $\alpha$  which was determined from the suggestions of Semple & Rigden (1984). In the sandy soils,  $f_s$  was taken to be 0.25 times the vertical effective overburden pressure. The values of  $f_s$  thus computed were generally about 20% larger than those determined from the Schmertmann-Nottingham correlations with cone sleeve resistance.
- (b) The end bearing resistance  $f_b$  was determined from the average cone penetration data within the vicinity of the pile tip. Values of 14 MN/m<sup>2</sup> and 2.45 MN/m<sup>2</sup> were determined for piles 1 and 2 respectively.
- (c) The initial tangent value of Young's modulus  $E_s$  of the clays was estimated to be 25 times the cone resistance. In the underlying dense sands, a value of  $E_s$  of 350 MN/m<sup>2</sup> was adopted. Poisson's ratio was taken as 0.5. The hyperbolic curve fitting factor  $R_f$  was taken as 0.9 for both shaft and base elements.
- (d) The pile modulus  $E_p$  was taken as 35000 MN/m<sup>2</sup>.



Fig. 52. Geotechnical data at Hemmant test pile site

A category 3B boundary element analysis was performed to predict the load-settlement behaviour, using the computer program PIES (Table 3).

Table 14 summarizes the Author's predictions of ultimate load, the measured values, and also the range of predictions made by the other participants. For both piles, the Author's prediction lay between the extremes, and was within  $\pm 20\%$ of the measured load. For pile 1, all 16 predictors underestimated the capacity, presumably because of underestimation of the shaft resistance  $f_s$ . Table 14 also summarizes the predicted and measured pile head stiffness values for pile 1. It is noticeable that the range of predictions is substantially greater than for the ultimate load predictions. Twelve of the 16 predictors, including the Author, underpredicted the pile head stiffness. For pile 2, the measured stiffness was similar to

	Load ca k	Pile head stiffness:† kN/mm		
	Pile 1 Pile 2		Pile 1	
Author's predictions	2490	1270	125	
Range of other participants' predictions (static analyses)	1090–3200	710–1400	49450	
Measured values	2080	1420	205	

Table 14. Summary of predicted and measured performance

\* At a deflexion of 50 mm.

† At a deflexion of 3.5 mm.



Fig. 53. Predicted and measured load-settlement behaviour pile 1

that of pile 1, and the Author and most of the predictors correctly predicted this characteristic.

Figure 53 shows the measured load-settlement curve for pile 1, the extreme predictions, and the Author's prediction. The most notable features of this figure are

- (a) the wide spread of predicted performance
- (b) the considerable difference between the Author's prediction and the measurements, despite the use of a site-specific category 3B analysis.

Following the revelation of the field measurements, the Author re-computed the performance of pile 1 using simple c legory 2 methods of calculation based on the parameters shown in Table 12. For calculation of ultimate load, the soil profile along the shaft was idealized as a 1 m zone of zero resistance, 3 m of soft clay, 4 m of loose sand, 13 m of soft clay, 5 m of stiff clay and 5.5 m of dense sand. The tip was assumed to also be in dense sand. The computed capacity in this case was 2420 kN, close to the initial predicted value. For settlement, the pile head stiffness was computed to be between 70 and 180 kN/mm, depending on whether the soil profile was taken broadly as a soft clay or a stiff clay. The average of these two values is 125 kN/mm, which corresponds to the Author's initial predicted value. Thus, for this case, rapid category 2 calculations, performed within about 10 min, give predictions which, although not in close agreement with the measurements, were no more inaccurate than the original category 3B predictions. This case study emphasizes once more that the method of calculation may play a far less significant role in pile performance prediction than does the idealization of the soil profile and the selection of the geotechnical parameters.

### CONCLUSIONS

This Paper has attempted to review the analysis of piles and pile groups under axial loading and to classify and place in perspective a number of existing methods of analysis. Several of these can be placed within the framework of the boundary element method, and can be used, at least in principle, to model many of the practical features of real piling problems.

A detailed examination has been made of some of the more significant aspects of the behaviour of single piles and pile groups, as revealed by the theoretical solutions. The problems considered include piles subjected to conventional static loading, to cyclic loading, and to loadings which arise from externally-imposed soil movements. Despite the simplifications inherent in the theory, there is compelling evidence to demonstrate that the behavioural characteristics revealed by the theory are consistent with observations made from field and laboratory tests on piles.

The importance of selecting appropriate geotechnical parameters has been emphasized, especially for problems that involve conventional loading. For practical application of most of the theories, it is generally necessary to make use of empirical correlations in order to obtain values of shaft resistance, end-bearing resistance and soil Young's modulus. A number of such correlations are summarized in the Paper.

By means of two case studies, an examination has been made of the sensitivity of pile performance predictions to factors over which the geotechnical analyst has some control, in particular, the method of analysis, the idealization of the soil profile and the values of the soil parameters used in the analyses. It is concluded that, while all these factors may have an influence, the latter two are generally of greater importance than the method of analysis.

A further case study has been described to demonstrate the difficulty of accurate prior prediction of pile performance, even in relatively simple geotechnical conditions in which a considerable amount of geotechnical data is available. In this case, much depends on the experience of the individual making the prediction, and the way in which he or she interprets the available data to obtain the required geotechnical parameters for the analysis employed.

The Author firmly believes that theoretical analyses have led to a significantly increased understanding of the mechanics of pile-soil interaction and an improved appreciation of the factors which influence pile behaviour. A most important feature of a soundly-based theoretical analysis is its ability to answer 'what-if' questions, for example, what if the pile length is increased, what if the number of piles is reduced, what if the load is cyclic rather than static? Theoretical analysis can provide a sound quantitative basis for answering such questions, provided that due care is taken in modelling the problem in hand. Indeed, the major challenge in applying theoretical approaches to practical pile design remains the proper characterization, idealization and geotechnical quantification of the site.

For most practical problems that involve conventional static loading of conventional-sized piles, there appears to be little justification for using very sophisticated methods of analysis, and quite often, category 2 solutions should provide an adequate basis for design. For a practical viewpoint, more refined category 3 analyses are only likely to be justified under the following circumstances

- (a) detailed design for major projects
- (b) unusually long or large-diameter piles
- (c) when detailed information is required on load transfer along the pile, or soil movements away from the piles are required
- (d) problems that involve unconventional loading, such as cyclic loading, or loading induced by movement of the soil past the pile.

From the point of view of gaining an increased understanding of pile behaviour, there is a definite need to pursue the development of more refined category 3 (especially 3C) analyses. A number of significant aspects of pile behaviour lack a proper theoretical framework of understanding, including

- (a) the influence of the method of installation on pile performance
- (b) the behaviour of piles in sand
- (c) creep deformation of piles
- (d) the mechanics of cyclic degradation of shaft resistance
- (e) the accumulation of permanent displacements under cyclic loading.

There is substantial scope for further research into such aspects so that those aspects of pile design which are still empirical may ultimately be replaced by more soundly-based approaches. The proper role of theoretical analysis in the development of improved pile design procedures has been well-stated by Terzaghi (1939)

To accomplish its mission in engineering, science must be assigned the role of a partner and not that of a master.

Theory (the science to which Terzaghi refers) should not be developed in isolation from controlled experiments and field observations, and should not be applied without due recognition of the importance of proper characterization of the site geology.

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### VOTE OF THANKS

### S. F. BROWN

Ladies and Gentlemen,

About 18 months ago, I wrote to Harry Poulos in my then capacity as BGS chairman inviting him to deliver the 1989 Rankine Lecture. As a consequence, I have carried a certain responsibility over this period, which is now happily discharged. My responsibility was enhanced by having had the unusual opportunity, with respect to a distant overseas lecturer, of briefing him at first hand when I attended the Australia-New Zealand Geomechanics Conference in Sydney in August 1988.

In my invitation to Harry, I said that we assumed the subject of piling could well feature in his presentation. Within two months, I had a letter outlining the lecture much as we heard it this evening. By the time we had lunch together in Sydney eight months later, all I had to do was make encouraging noises, since it was clear to me that a balanced, well planned and extremely appropriate lecture was in prospect. I took the precaution of telling Harry how even greater men had found the Rankine Lecture occasion a difficult one, that the audience varied from the least to the greatest in British Geotechnics, and that all would wish to go away having learned something. I mentioned visual aids en passant remembering some past examples of slides that left something to be desired.

Towards the end of the Sydney Conference, I had the further opportunity of listening to Harry Poulos eloquently deliver the John Jaeger Memorial Address on the subject of calcareous soils. By then no doubts remained.

Against this background, therefore, and in common with many others who had been privileged to hear Harry Poulos speak on other occasions, I arrived this evening, not merely with the usual anticipation of the big event, but with the confidence that we were to hear an excellent lecture presented with clarity and authority. Ladies and Gentlemen, we have not been disappointed.

Harry, you have demonstrated vividly the use which can be made of what Terzaghi somewhat scornfully referred to as 'Science' and you have put in clear perspective the other less prudent statement he made with Ralph Peck that theoretical refinements for piles are completely out of place. Your balanced approach giving a sense of perspective on the input parameters for design through your sensitivity analyses and presenting applications of theory through case studies and a class A prediction have clearly demonstrated how theory should be used, not as a master but as an essential tool in geotechnical engineering.

Professor Poulos, the British Geotechnical Society is enormously grateful to you for preparing such an excellent and stimulating Rankine Lecture, which was full of useful practical information and will, I predict, become a standard reference when it appears in *Géotechnique* in due course.

We thank you and admire you for delivering this lecture with such EASE—eloquence, aplomb, stamina and enthusiasm.